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THEORETICAL EVALUATION OF THERMAL ELASTO- PLASTIC DAMPING COEFFICIENT IN BEAMS OF RECTANGULAR CROSS SECTION

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COMMITTE DECISION

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NOMENCLATURE

A : Cross sectional area of the beam.

C : Specific heat per unit mass.

c : Hight to length ratio of the beam

C_{ijkl} : Tensor of elastic contants.

C_v : Specific heat per unit mass.

D_{ij} : Rate of deformation tensor.

ds : Increase of entropy.

E : Young's modulas of elasticity.

$\dot{e}^p_{ij}$: Rate of change of plastic strain tensor.

F : Applied force

$F(\sigma_{ij})$: Yield function.

G : Shear modulas.

G_β : Thermal damping function

h : Hight of the beam.

I : Area moment of inertia.

i, j, k : Indicies each have a range of 1, 2, 3

J : Stress deviation with respect to the mean stress.

K : Curvature of the beam.

Ko : Initial curvature of the beam.

L : Length of the beam.

$M(x,t)$: Bending moment

m : Mass.

$N(x,t)$: Axial force.

$\bar{N}^*$: Normalized temperature.

n : Number determines the mode shape.

No : Magnitude of $\bar{N}^*$.

q_i : Heat flux vector.

Sp : Entropy per unit mass.

To : Equillibrium temperature

Y : Dimentionless Coordinate.

Greak Symbols:

δ_{ij} : Kronecker delta function.

ν : Poisson's ratio.

Ψ : Volume averaged damping of a structure.

Ψ_L : Local specific damping.

σ_{ij} : Stress tensor.

ω : Circular frequency.

ω_0 : Reference frequency

τ : Characteristic time.

Ψ_0 : Characteristic damping coefficient.

α : Coefficient of thermal expansion.

ρ : Density - (mass per unit volume).

θ : Temperature.

Ω : Normalized frequency.

Φ : phase of $\bar{N}^*$.

β : Thermomechanical coupling parameter.

ϵ_{ij} : Strain tensor.

ε : Internal energy per unit mass.

μ : Leme's Constant.

ξ : Normalized coordinate in x-direction.

η : Normalized coordinate in y-direction.

λ : Complex frequency = $i\omega$

ABSTRACT

Theoretical Evaluation of Thermoelastoplastic Damping Coefficient in Beams of Rectangular Cross Section.

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In this work the thermoelastoplastic damping coefficient and thermomechanical coupling function of beams with rectangular cross section undergoing free transverse vibration have been studied based on the elementary principles of elasticity, plasticity and thermodynamics[1,2]. The first and second law of thermodynamics were taken as a starting point to develope the general theory of thermal damping in elastic - perfectly plastic solids. According to the known Thomson effect [3] which takes place in a solid medium an irreversible heat transfer and entropy is created due to an applied stress field. This entropy is a measure of the amount of work that is converted into heat. i.e mechanical damping. By way of illustration, solution of the problem of flexural vibration of a Bernoulli - Euler beam subjected to adiabatic boundary conditions and the magnitude of the thermal damping coefficient was calculated. General formulation of the vibration boundary value problem and thermoelastoplastic damping coefficient for rectangular cross section beams under general mechanical boundary condition and the thermal boundary conditions that follows Newton's heat exchange law were considered.

Solution of the transverse vibration eigenvalue problem is presented for the general case of boundary conditions. Special considerations were made to calculate the magnitude of the thermal damping coefficient in a closed form for the following three types of thermal boundary conditions:

(1) All surfaces of the beam are thermally insulated.

(2) All surfaces of the beam are kept at constant temperature.

(3) Lateral surfaces are thermally insulated and the end surfaces are kept at constant temperature.

Numerical results for thermal damping coefficients of Bernoulli-Euler beams for the above three cases were presented in a graphical form and compared with those of Zener and [3,4]. It was found that the present results differ very slightly from those in [3,4] if the value of nc $(nc = \frac{1}{g_0}, g_0 =$ wave length Parameter) of the beam is less than 0.1, but the difference becomes more important as this ratio increases from 0.1 to a larger value. It was also observed that the peak values of the calculated and the approximate damping coefficients are almost identical compared with those reported in [3,4] for all values of wave length parameter ratio g_0.

The magnitude of the thermomechanical coupling functions for the above mentioned three cases were obtained in a closed form, also a general formulation for the heat conduction equation in elastic perfectly plastic medium were developed and the magnitude of the thermal damping coefficients were obtained and presented in graphical forms.

CHAPTER ONE
INTRODUCTION AND LITERATURE REVIEW
1.1 INTRODUCTION

It is a well known fact that beams represent one of the most important structural members in engineering design and construction. For a given engineering structure, several methods based on the well developed theories of dynamics and elasticity are available to the analyst for a reasonably accurate evaluation of the inertia and stiffness properties of the structure. Although, damping is present in all oscillatory systems only "rough" approximate analytical and experimental methods are available for estimating the damping in a given vibrating structure. The effect of damping is to remove energy from the system. Energy in a vibrating system is either dissipated into heat or radiated a way. Dissipation of energy into heat can be experienced simply by bending a piece of metal back and forth a number of times.

In vibration analysis we are concerned with damping present in the system, as the loss of energy from a vibrating system results in the decrease of vibration amplitude. A vibrating system may encounter many different types of damping forces from internal molecular friction to sliding friction and fluid resistance, etc. Generally the mathematical description of the damping forces is quite complicated.

Energy dissipation is always determined under conditions of cyclic oscillation. Depending on the type of the damping present the force displacement relationship when plotted may differ greatly. In all cases, however the force displacement curve will enclose an area, referred to as the hysteresis loop that is proportional to the energy lost per cycle. In this case the area of hysteresis loop corresponds to the energy spent to produce depends on both mechanical and thermal properties of the material such as E, α, To, ρ, c.

Due to its importance in the analysis of elastic systems and viscoelastic systems subjected to conservative and nonconservative forces, the thermomechanical coupling effect has received considerable attention[5,6,8] because thermomechanical coupling is always present in any real dynamic system. This coupling modifies the stiffness of the elastic system and contributes thermal damping to the system. Experimental results show that the material damping in the elastic range for a wide class of metals especailly aluminum is almost entirely caused by thermomechanical coupling effect[4].

Thermal damping is another kind of damping presents inside the material itself due to the existence of the temperature gradient between different parts of the material - due to the contineous heating of the compressional side and cooling of the tensile side of the material undergoing either transverse or longitudinal harmonic vibration.

This means that thermal damping is due to thermal currents within the material which is accompanied by very small loss of stiffness and strength where entropy is created due to the heat flow a cross the temperature gradient and consequently, work (mechanical energy) is converted into heat. Therefore one expects that thermal damping arises in many practical engineering systems, such as bridges and many other vibrated engineering structures.

A review of the literature seems to indicate that although a few investigations of stress and strain analysis of elastic-plastic beams at elevated temperatures have been made in the past [8-23], very little has been done on the dynamic analysis of thermally induced motion of elastic plastic beams. Published theoretical and experimental works dealing with thermal damping phenomenon are still limited. The existing experimental and analytical results indicate that material damping in elastic range of deformation for a large number of metals can be analyszed within the framework of the theories of coupled linear thermoelasticity. Although thermal damping is usually small it has an important effect in the study of dynamic instablity of elastic continueous systems subjected to nonconservative forces-and also in the study of thermal balancing of rotating shafts.

The first study of the thermal damping mechanism was done by Zener [3,4] 1937 based on the linear theory of thermoelasticity. His work did not take into account the effect of longitudinal heat flows [induced heat flow due to the existance of thermomechanical function].

In recent studies the governing equations for the free - vibration boundary value problem of a simple rectangular cross section beam subjected to general mechanical boundary conditions and thermal boundary conditions that follows Newton's surface heat exchange law have been derived [4]. These studies were done within the framework of coupled linear thermoelasticity theory. Fung [1] theoretically presented the main governing equations connecting thermal and mechanical properties of solids. He derived the main relations which connect the specific heat, the modulas of elasticity, the latent heat of change of strain and stress at variable temperatures. Also the relationship between the rate of change of temperature and strain and the ratio of adiabatic to isothermal elastic moduli were studied. Mase and George [2] presented the contribution of stress field tensor which can be divided into two parts: first stress field tensor and second temperature field tensor.

The concept of using the second law of thermodynamics as a starting point to develope a general theory to calculate thermal damping in solids was studied by kinra and Milligan[3]. Based on this theory the applied stress field is viewed as the cause which results in temperature field. This will cause irreversible heat to transfer and entropy is created as a result thereof. In their work the effect of longitudinal heat flow and the isothermal boundary conditions were neglected and the problem is solved only for Bernoulli-Euler beam corresponding to adiabatic boundary conditions.

The study of thermal damping is of great important in the study of thermal balancing. i.e it is used in on-line thermal balancing techniques for a large turbo-generator [8]. This technique is proposed to rebalance automatically a large turbogenerator during operation. The sensitivity of the rotor unbalances to thermal asymmetries in the rotor is exploited by mounting some heating elements and using them as controlled actuators.

Until now the thermal balancing of a large rotor is very rare procedure, and is only done in some specail cases like the static thermal balancing, which is done by adjusting the ventillation in the rotor in order to reduce cooling locally.

When the rotor is rotated the bearing reactions of the rotor are caused by thermally induced unbalances as well as other unpredictable unbalances. Coupled thermoelastic heat conduction equation for the rotor should be developed and therefore thermoelastic damping is presented during operation.

1.2 LITERATURE SURVEY

A limited number of studies dealing with the calculations of thermoelastic damping coefficient have been made, for both circular and rectangular cross section beams [4,5]. The mathematical formulations describing the behavior of elastic solid media under the combined action of heat and external loads are examined by different investigators [18-47]. These studies make use of basic concepts of mechanics and thermodynamics which underline the behaviour of a contineous media..

In recent studies the governing equations for the free - vibration boundary value problem of a simple rectangular cross section beam subjected to general mechanical boundary conditions and thermal boundary conditions that follows Newton's surface heat exchange law have been derived [4]. These studies were done within the framework of coupled linear thermoelasticity theory. Fung [1] theoretically presented the main governing equations connecting thermal and mechanical properties of solids. He derived the main relations which connect the specific heat, the modulas of elasticity, the latent heat of change of strain and stress at variable temperatures. Also the relationship between the rate of change of temperature and strain and the ratio of adiabatic to isothermal elastic moduli were studies. Mase [2] presented the contribution of stress field tensor which can be divided into two parts: first stress field tensor and second temperature field tensor.

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It is well known that there exists a relationship between the loss of energy and the maximum stress amplitude. This may be used for estimating the material damping in a cantilever beam [5]. This concept has been used to estimate material damping in terms of stress distribution functions for each mode of vibration and damping stress functions for each mode of vibration which is influenced by temperature gradient [5].

In these works the effect of the thermal stress field for each mode of vibration and the longitudinal heat flow and the effect of the angular rotation of a generic beam cross section, and the thermal shear force and the thermal bending moment were neglected.

A general method to study thermally induced vibrations of viscoelastic plate of circular cross section is proposed by Mazumdar [47]. It is found that the time behaviour of the plate can be found by assuming normal mode expansion in terms of eignfunction for the associated elastic plate problem, and the deflection is obtained by using elastic-viscoelastic analogy. Their results shows that for values of β (thermal coupling parameter) less than 0.017 the plate undergoes damped oscillation.

In this work the general theory of thermoelasticity[1,2] was extended to calculate the magnitude of thermal damping coefficient in elastic perfectly plastic solids, also a general formulation of the heat conduction equation in elastic plastic medium was developed and solved under general mechanical boundary conditions and thermal boundary conditions that follows Newton's heat exchange law. The magnitude the thermal damping coefficient as function of frequency parameter for Bernoulli-Eulear beam was obtained in a closed form for the last mentioned thermal boundary conditions and compared with that of Zener. [3,4].

1-3 OBJECTIVES OF THE THESIS

In light of the above introduction, the main objectives of the present work can be summarized as follows:

1- To formulate the general theory for the free vibration boundary value problem of a rectangular cross section beam under general mechanical boundary conditions and thermal boundary conditions that follows Newton surface heat transfer law based on the general theory of linear thermoelasticity.

2- To calculate the thermal damping coefficient either in elastic or elastic perfectly palastic medium for simply supported beam of rectangular cross section in the following cases:

a. All surfaces are kept at constant temperature (i.e isothermal boundary conditions corresponding to small frequency and small heat generated inside the beam).

b. All surfaces of the beam are thermally insulated. i.e. corresponding to a large frequency and high heat generated inside the beam.

c. lateral surfaces of the beam are thermally insulated and end surfaces are kept at constant temperature.

3. To evaluate the thermomechanical coupling function of the beam under consideration subjected to the adiabatic and isothermal boundary conditions.

4. To make a comparison between the above thermal damping coefficient functions.

5. To study the relation between the main parameters affecting the thermal damping function such as normalized temperature, normalized frequency and normalized coordinates.

6. To campare the above results with Zener's results.

1.4 ORGANIZATION OF THE THESIS

The thesis is divided into seven chapters. In chapter one the general definitions of damping and thermal damping was introduced. Chapter two gives a review of the main physical concepts and definitions needed to formulate the thermal damping mechanism in the elastic ranges of deformation.

In chapter three and four the coupled heat conduction equation in elastic perfectly plastic medium is developed and solved corresponding to adiabatic boundary conditions and the magnitude of the volume averaged thermal damping coefficient is obtained in a closed form.

In chapter five a general formulation of the vibration boundary value problem and the governing equations for the free vibration boundary value problem under general mechanical and thermal boundary conditions are presented.

Then in chapter six the magnitude of the thermomechanical coupling functions and the thermal damping coefficients are obtained in a closed form. Finally, chapter seven is devoted to the conclusions and recommendations which may be useful for furture investigations.

where μ is the coefficient of friction, N is the normal force once the motion is initiated.

3- **Structural damping:** This damping results when materials are cyclicly stressed. Thus energy is dissipated internally within the material itself and may be written as

$$Wd = \alpha X^2 \quad \text{............ (2-1.3)}$$

where

Wd: energy dissipated per cycle.

α : a constant of proportionality

X^2 : the square of the amplitude of vibration.

The effect of damping is to remove energy from the system. Energy in a vibrating system is either dissipated into heat or radiated away. In vibration analysis, we are concerned with damping in terms of system response. The loss of energy will result in a decay of amplitude of free vibration. Energy dissipation is determined under conditions of cyclic oscillations. In all cases, the force - displacement relationship (curve) will enclose inside itself an area known as hysteresis loop, that is proportional to the energy lost per cycle. The energy lost per cycle (Wd) due to a damping force Fd is given by

$$W_d = \oint F_d . dx \quad \text{........ (2-1.4)}$$

In general W_d depends on many factors such as temperature and frequency.

2-2 THOMSON EFFECT AND THERMAL DAMPING:

Thomson effect says that when a thermoelastic solid material is subjected to a axial stress it cools and the compressed side is heated. This means that temperature gradient exist inside the material. Also when a homogeneous material is subjected to any stress field, homogeneous or inhomogeneous different parts of the material undergo different temperature changes. The existing temperature gradient will result in irreverisible heat conduction inside the material parts. Thus entropy is created due to the heat flows across a temperature gradient and consequently work is converted into heat. This process is the origin of thermoelastoplastic damping process. Thermal damping is therefore in addition to all other sources of material damping present inside the material itself.

Thermomechanical coupling effect in the analysis of elastic systems and viscoelastic systems subjected to conservative and nonconservative forces, received considerable attentions because thermomechanical coupling is always present in any real dynamic system [23, 24, 25]. This coupling slightly modifies the stiffness of the elastic system and contributes thermal damping to the system. Experimental result shows that the material damping in the elastic range for a wide class of metals especailly aluminum is almost entirely caused by thermomechanical coupling effect.

However, the deformations due to the external loads are always accompanied only by small changes in temperature[23-33], and therefore it is sometimes reasonable to calculate these deformations without taking into account the effect of thermal expansion. Similarly, if strains are produced in a body by a nonuniform temperature distribution, it would seem intuitively clear that the influence of these strains on the temperature itself should not be large. In actual cases coupling term between heat and strains can't be disregarded for many kind of problems especailly those in which the thermoelastic dissipation is of primary interest.

It should be noted that through thermomechanical coupling, a portion of kinetic energy of the vibrated system is converted into heat energy, which causes temperature gradient inside the solid bodies and therefore produces material damping of thermoelastic nature. When a material is subjected to a variable strain, many things happen inside which are not explicitly observed. For example when a metal is uniformely strained in a macroscopic sense, the individual anisotropic crystals are strained differently and thermal currents that circulate among the crystals are generated; however, the interstitial atoms move in a considered crystals or among the crystals.

Coupled linear thermoelasticity and plasticity theory can be used to predict material damping in elastic and plastic range of deformation and coupled one way heat conduction equation can be used to predict material damping in plastic range of deformation. Although this thermal damping is usually small, it has an important effect of the dynamic instabillity of elastic contineous systems (solid bodies) subjected to conservative or nonconservative forces and also in thermal balancing study. The remainder of this chapter briefly outlines the main physical concepts and laws of the stress - strain and the internal energy of the solid material which will be used later to evaluate the magnitude of thermal damping coefficient in a vibrating elastic - plastic structure. More indepth discusions of concepts and laws are given in the indicated references

2-3 SMALL DEFORMATION THEORY-INFINITESIMAL STRAIN TENSORS: In this section the relations between infinitesimal strain tensor and the displacement vector are developed based on the principles of continuum mechanics. The so-called small deformation theory of continuum mechanics has its basic conditions, the requirement that the displacement gradients be small compared to unity. The Eulerian finite strrain tensor ϵ_{ij} has the form[1,2]

$$\epsilon_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}-u_{k,i}.u_{k,j}\right) \quad ....(2\text{-}3.1)$$

where u is the displacement vector. If the displacement gradients are small the finite strain tensors in equation (2-3.1) reduce to infinitesimal strain tensors, and the resulting equations represent small deformations. In equation (2-3.1) if the displacement gradient components $u_{k,i}, u_{k,j}$ are each small, the product terms are negligible and may be dropped, and the resulting tensor is the Eulerian infinitesimal strain tensor ϵ_{ij} which is denoted by

$$\epsilon_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right) \quad .... (2\text{-}3.2)$$

This equation will be used later in to develope the coupled heat conduction equation in elastic - plastic structure.

2-4 LINEAR ELASTICITY: HOOK'S LAW AND STRAIN ENERGY FUNCTION

In this section the main relations between stress and strain energy function are presented based on the generalized Hookes Law. In linear elasticity theory it is assumed that displacements and displacement gradients are sufficiently small so that no distinction need to be made between the Lagrangian and Eulerian descriptions. The linear strain tensor is given by[1, 2]

$$\epsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}) \quad ....(2\text{-}4.1)$$

Assuming that the deformation processes are adiabatic (no heat loss or gain) and isothermal (constant temperature), the constitutive equations for linear elastic solid relate the stress and strain tensors are given by the generalized Hooke's law[1,2]

$$\sigma_{ij}=C_{ijkm}\cdot\epsilon_{km} \quad .... (2\text{-}4.2)$$

where Cijkm is a tensor of elastic constants, σ_{ij} is the stress tensor, and ϵ_{km} is strain tensor. When thermal effects are neglected the energy balance or first law of thermodynamics may be written as[1,2,6]

$$\frac{d\varepsilon}{dt} = \frac{1}{\rho}\sigma_{ij}\ \dot{\epsilon}_{ij} \qquad \text{.................... (2-4.3)}$$

where ε is the internal energy per unit mass and ρ is the mass density per unit volume. The internal energy in this case is purely mechanical and is called the strain energy (per unit mass).

Let the strain energy per unit volume defined as $\bar{\varepsilon}$, $\bar{\varepsilon} = \rho\varepsilon$, then from above one has

$$\bar{\varepsilon} = \frac{1}{2}C_{ijkm}\ \epsilon_{ij}\ \epsilon_{km} \qquad \text{....... (2-4.4)}$$

or

$$\bar{\varepsilon} = \frac{1}{2}\sigma_{ij}\ \epsilon_{ij} \qquad \text{..................(2-4.5)}$$

From equation (2-4.5) one can calculate the magnitude of the strain energy if the stress and the strain tensors are known.

2-5 ISOTROPIC MEDIA AND ELASTIC CONSTANTS

In this section Hooke's law in terms of elastic constant is developed for an isotropic media, and the main relations between elastic constants are summarized. Bodies which are elastically equivellent in all directions possess a complete symmetry are termed isotropic. For isotropic materials the number of elastic constants reduces to 2, these are E and G, and the matrix of elastic constants is symmetric regardless of existence of a strain energy function.

Hooke's law for isotropic body is written in the form[1,7]

$$\sigma_{ij} = \lambda\,\delta_{ij}\ \epsilon_{kk} + 2\mu\ \epsilon_{ij} \qquad \text{.........(2-5.1)}$$

where λ and μ are are lame's constants which depend on the modulas of elasticity of the material E and possion's ratio ν. For a simple uniaxial state of stress in $\bar{x}$ direction, engineering constants E and ν may be used through the relationships

$$\sigma_{xx} = \epsilon_{xx}.E \qquad \text{............ (2.5.2)}$$

$$\epsilon_{yy} = \epsilon_{zz} = -\nu\ \epsilon_{xx} \qquad \text{.................(2.5.3)}$$

In terms of these elastic constants Hooke's law equation (2-5.1) for isotropic bodies becomes:

$$\sigma_{ij} = \frac{\epsilon}{1+\nu}\left(\epsilon_{ij} + \frac{\nu}{1-2\nu}\delta_{ij}\,\epsilon_{kk}\right) \text{............. (2-5.4)}$$

Note that if **k** is the bulk modulas and G is shear modulas then,

$$k = \frac{E}{3(1-2\nu)} = \frac{3\lambda + 2\mu}{3}\ ;\ \mu = G = \frac{E}{2(1-\nu)} \text{ ....(2-5.5)}$$

Equation (2-5.4) describes the relation between the stress and the strain for an istropic elastic material.

2-6 HOOKE'S LAW INCLUDING THE EFFECT OF THERMAL EXPANSION:

Based on the elementary principles of thermoelasticity a generalized form of Hooke's law including the effect of thermal expansion are developed. When thermal effects are taken into account, the components of linear strain tensor ϵ_{ij} may be assumed to be the sum[2].

$$\epsilon_{ij} = \epsilon_{ij}^{(s)} + \epsilon_{ij}^{(T)} \qquad \text{................. (2-6.1)}$$

where $\epsilon_{ij}^{(s)}$ is the contribution from stress field and $E_{ij}^{(T)}$ is the contribution from temperature field. $\epsilon_{ij}^{(T)}$ may be assumed to take the linear form[1,14]

$$\epsilon_{ij}^{(T)} = \alpha\ (T\text{-}To)\ \delta_{ij} \qquad \text{......................(2-6.2)}$$

where α is the linear coefficient of thermal expansion, T_0 is the reference temperature and δ is the kronecker delta function. Substituting equation (2-6.2) into equation (2-6.1) and using equations of isotropic media leads to[14,18]

$$\epsilon_{ij} = \frac{1}{2\mu}\left(\sigma_{ij} - \frac{\lambda}{3\lambda + 2\mu} \cdot \delta_{ij}\,\sigma_{kk}) + \alpha(T - To)\delta_{ij}\right) \qquad (2\text{-}6.3)$$

or

$$\sigma_{ij} = \lambda\delta_{ij}\;\epsilon_{kk} + 2\mu\;\epsilon_{ij}\,(T - T_o) \qquad (2\text{-}6.4)$$

This equation relates stress to strain in elastic solids taking into consideration the effect of thermal expansion due to the applied stress field. This equation will be used later in chapter (3) to develope the coupled heat conduction equation in elastic solids, which is known as Duhamel - Neumann relations[2].

2-7 BASIC CONCEPTS AND DEFINITIONS OF PLASTICITY

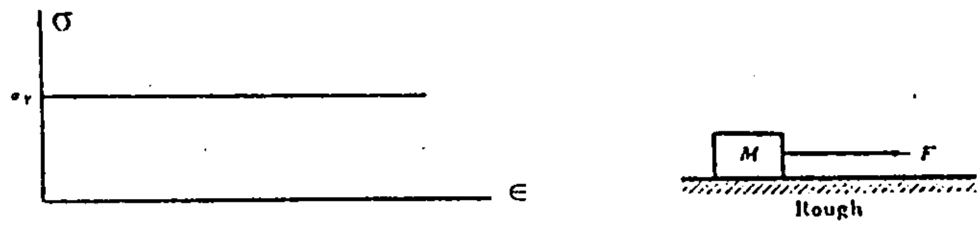
In this section to section (2-13) the main basic assumptions and definitions of the mathematical theory of plasticity are summarized. Elastic deformations are characterized by complete recovery to the undeformed configuration upon removal of the applied loads. The plastic deformations depends upon the stress magnitude and not upon the straining or loading history. In particular, deformations which results from the mechanism of slip, or from dislocations at the atomic level, and which thereby lead to permanent dimensional changes are known as plastic deformations.

The primary concerns here are with the mathematical formulation of the stress-strain relationships suitable for the description of plastic deformation, and with the establishment of appropriate yield criteria for predicting the onset of plastic behavior.

Allthough it is recognized that temperature will have a definite influence upon the plastic behavior of real material, it is customary in much of plasticity to assume isothermal conditions and consider temperature as a parameter. Aso it is common practice in plasticity to neglect any effect that the rate of loading would have upon the stress strain curve.

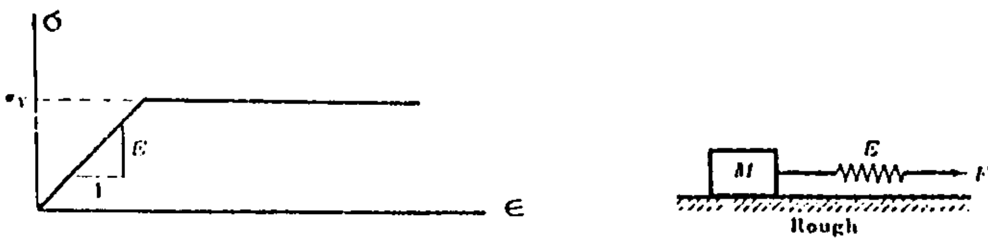
Most of the theories for analyzing plastic behavior may be looked upon as a generalization of certain idealization of one dimensional stress strain curve. Two of the most commonly used models for the idealization of stress-strain relations in plasticity are the two shown in Figure (2-7.1) along with a simple mechanical model for each. In the models the displacement of the mass M depicts the plastic deformation and the force F plays a stress role.

(a) Rigid - perfectly plastic



(a) Rigid – Perfectly Plastic

(b) Elastic - perfectly plastic



(b) Elastic – Perfectly Plastic

Fig. [2-7.1]

In Fig. 2-7.1a, elastic response and work hardening are missing entirely, whereas in (b), elastic response prior to yield is included. In the absence of work hardening the plastic response is termed as perfectly plastic.

2-8 IDEAL PLASTICITY:

We can define ideal plasticity as plastic deformation without strain hardening. Work hardening in a simple test means that the stress is a monotonically increasing function of increasing strain. In general, work hardening means that for all added sets of stresses a positive work is done by external agency during the application of the stresses, and the net work performed by it over the cycle of application and removal is either zero or positive.

Assume that a yield function $f(\sigma_{ij})$ exists, which is a function of the stresses σ_{ij} only and not dependent on plastic strain $e_{ij}^{(p)}$ then the rate of change of plastic strain tensor $e_{ij}^{(p)}$ is given by[1]:

$$\dot{e}_{ij}^{(p)} = \frac{1}{\mu} \frac{\partial f}{\partial \sigma_{ij}} \qquad \text{.............. (2-8.1)}$$

where μ has the significance of the coefficient of viscosity. The sign of μ is restricted by the condition that plastic flow always involves dissipation of mechanical energy $\dot{W}$, defined as:

$$\dot{W} = \sigma_{ij} \, \dot{e}_{ij}^{(p)} > 0 \qquad \text{............(2-8.2)}$$

Using Von Mises yielding condition one gets [1]

$$f(\sigma_{ij}) = J_2 - k^2 \quad . \qquad \text{............(2-8.3)}$$

where $J_2 = \frac{1}{2}\sigma'_{ij} \bullet \sigma'_{ij}$, k = constant and σ'_{ij} is the stress deviation tensor.

Substituting equation (2-8-3) into (2-8.1) yields.

$$\dot{e}_{ij}^{(p)} = \frac{1}{\mu} \frac{\partial f}{\partial \sigma_{ij}} = \lambda \, \sigma'_{ij} \qquad \text{.............(2-8.4)}$$

This equation relates the rate of plastic strain tensor to the stress deviation tensor depending on Von-Mises criteria. This equation will be used later in chapter (4) to develope the coupled heat conduction equation in plastic solids.

2-9. FIRST LAW OF THERMODYNAMICS:

A brief summary of the basic structure of general theory of thermodynamics is given in this section. Only closed systems, i.e systems which do not exchange matter with their surroundings, are considered here. Also the system is to be assumed isolated, i.e no interactions between the system and its surroundings.

A system surrounded by an insulator is said to be thermally insulated, and any process taking place inside this system is called adiabatic. A system is said to be homogeneous if the state variables not depend on space coordinate.

The first Law of thermodynamics can be formulated as follows. If a thermally insulated system can be taken from a state I to a state II by alternative paths, the work done on the system has the same value for every such (adiabatic) path. This means that the increase of energy, for any adiabatic process is equal to the work done on the system. Thus

$$\Delta \text{ energy} = \text{work done } (W) \quad \text{(adiabatic process)} \qquad ....(2\text{-}9.1)$$

Now define the heat Q absorbed by a system as the increase in energy of the system less the work (W) done on the system. Thus,

$$Q = \Delta \text{ energy} - \text{work done} \qquad .... (2\text{-}9.2)$$

or

$$\Delta \text{ energy} = Q + \text{work done} \quad \text{(all processes)}. \qquad ....(2\text{-}9.3)$$

Then by comparing of equation (2-9.1) with (2-9.2) one observes that the energy of a system can be increased either by work done on it or by absorption of heat. The total energy is made up of kinetic energy k, gravitational energy G, and internal energy E.

If both mechanical and non-mechanical energies are to be considered, the principle of conservation of energy in its general form must be used. In this form the conservation principle states that the time rate of change of the kinetic plus the internal energy is equal to the sum of the rate of work plus all other energies supplied to or removed from the continuum per unit time. Therefore the energy principle for a thermomechanical continuum is given by[1,2]

$$\dot{K} + \dot{U} = \dot{M} + \dot{Q} \qquad \text{............(2-9.4)}$$

where K is the kientic energy of the system, U is the intemal energy, M is the mechanical energy, Q is the heat added or removed from the system. The general form of the first law of thermodynamics for a thermomechanical continuum can be written in the form [1,2]

$$\rho\dot{\varepsilon} = \sigma_{ij}\,\dot{\epsilon}_{ij} + q_{i,i} + \rho h \qquad \text{................(2-9-5)}$$

where $\dot{\varepsilon}$ is the internal energy per unit mass, ρ is the mass density of the solid medium, σ_{ij} is the stress teneor, $\dot{\epsilon}_{ij}$ is rate of deformation tensor, q is the heat flux, h is the heat supply per unit mass. Noting that this equation represents the first law of thermodynamics for elastic solids and will be used later to develop heat conduction equation in elastic solids.

2-10 LINEAR MOMENTUM PRINCIPLE AND EQUATIONS OF MOTION

A moving continuum which occupies a volume Ω at time t with body forces F per unit mass has a linear momentum L defined as the velocity vectorV summed over incremental mass dm or as the product of a velocity vector with density summed over incremental volume:(i.e)

$$L = \int_m v\,dm = \int_\Omega v\,\rho\,d\Omega \qquad \text{.................... (2-10.1)}$$

Nothing that $\rho\, d\,\Omega$ = constant i.e $(\rho_0\, d\Omega_0 = \rho\, d\Omega)$

$$\frac{d}{dt}\left(\rho_0\, d\Omega_0\right) = \frac{d}{dt}(\rho\, d\Omega) = 0 \qquad \text{........................ (2-10.2)}$$

And assuming that the stress tensor is symmetric, we get the equation of motion of the undeformed cartesian coordinates in the form [1,2,3].

$$\sigma_{ij,j} + \rho\, F_i = \rho \ddot{u}_i \qquad \text{......... (2-10.3)}$$

where σ_{ij} is the stress tensor and u_i is the displacement vector. It is to be noted that this equation will be used later in chapters (3) and (4) to develope general theory of thermal damping in elastic-perfectly plastic solids.

2-11. ENTROPY AND SECOND LAW OF THERMODYNAMICS

The second Law of thermodynamics postulates the existence of two distinct state functions; T the absolute temperature, and S the entropy with certain properties:

1. T is a positive number which is a function of temperature.
2. The entropy of a system is equal to the entropies of its parts.

3. The entropy of a system can change into distinct ways, namely, by interaction with the surroundings and by changes taking place inside the system. We may write

$$dS = dS_e + dS_i \qquad \text{........ (2-11.1)}$$

where dS denotes the increase of entropy of the system, dSe denotes the part of this increase due to interaction with the surroundings, and dsi denotes the part of this increase due to changes taking place inside the system.

The term dSi is zero for reversible process.

$$dSi > o \qquad \text{irreversible process} \qquad \text{......... (2-11.2)}$$

$$dSi = o \qquad \text{reversible process} \qquad \text{............ (2-11.3)}$$

In a reversible process, if dq(R) denotes the heat supplied per unit mass to the system, the change dSe is given by

$$dSe = \frac{dq(R)}{T} \qquad \text{(reversible process) .... (2-11.4)}$$

2-12. HEAT CONDUCTION

The entropy production per unit volume are developed in this section based on the elementary principles of heat transfer in a solid bar. Consider the heat transfer in a slender solid bar with continuous temperature gradient in the direction of the lengthwise axis of the bar ,x , as shown in Fig. 2-12.1 The tempenture is assumed to be uniform in each cross section of the bar and the walls except the ends are thermally insulated. Furthermore it is assumed that the bar is free of stresses.

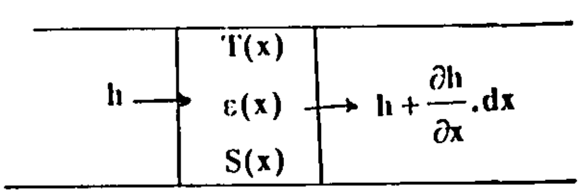


Fig. [2-12.1] Heat conduction

Let $T(x)$, $\varepsilon(x)$, $S(x)$ denote, respectively, the temperature, the internal energy per unit mass, and the entropy per unit mass of the solid. Let h denote the heat flux per unit area per unit time in the x- direction. If Q represents the heat transported to the right across a unit cross-sectional area, then

$$h = \frac{dQ}{dt} \qquad \text{.......... (2-12.1)}$$

Now consider the changes of heat occuring in a small element of length dx in a small time interval dt. The net increment of heat in this element is

$$dQ = hdt - (h + h_{,i}.dx_i).dt = -h_{,i}.dx_i.dt \qquad \text{......... (2-12.2)}$$

Based on the basic concepts of thermodynamics and heat transfer the entropy flow may be defined as h/T, and the entropy production per unit volume is defined as

$$\rho\frac{ds}{dt} = -\frac{h}{T^2}T_{,i} \qquad \text{................ (2-12.3)}$$

Generalizing the above results to the three-dimensional case, the entropy production per unit volume is

$$\rho\,\frac{ds}{dt} = -\left[\frac{h_x}{T^2}\frac{\partial T}{\partial x} + \frac{h_y}{T^2}\frac{\partial T}{\partial y} + \frac{h_z}{T^2}\frac{\partial T}{\partial z}\right]$$

$$= -\frac{h\,T_{,i}}{T^2} \qquad \text{........... (2-12.4)}$$

where hx, hy, hz are the three components of the heat flux vector h, i.e hx is the heat flux per unit area across a surface element normal to the x-axis, ect. It should be noted that the above equation will be used in section (3-5) and (4-5) to calculate the temperature gradient in elastic - perfectly plastic .

solids and then to calculate the magnitude of the thermal damping coefficient.

2-13 EULER EQUATION FOR BEAMS

To determine the lateral vibration of beams, consider the forces and moments acting on an element of the beam shown below.

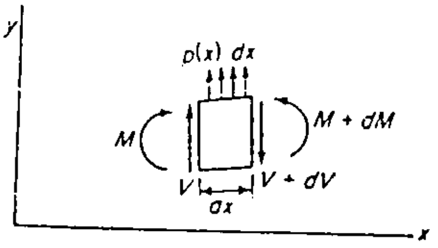


Figure [2-13.1]

Where V and M are shear and bending moments, respectively, and p(x) is the Loading per unit Length of the beam. Summing forces in y - direction yielding

$$dv - p(x)\,dx = o \qquad ....(2.13.1)$$

Summing moments about any point on right face of the element

$$dM - v dx - \frac{1}{2} p(x)(dx)^2 = o \qquad ....(2.13.2)$$

Also

$$\frac{dM}{dx} = V \quad , \frac{d^2M}{dx2} = \frac{dv}{dx} = p(x) \qquad .... (2\text{-}13.3)$$

The bending moment is related to the curvature by flexure equation, according to Bernoulli - Euler beam theory [16]

$$M = EI \frac{d^2y}{d\,x2} \qquad .... (2\text{-}13.4)$$

Substituting equation (2-13.5) into (2-13.4) one get

$$\frac{d^2}{dx2}\left(EI\frac{d^2y}{dx2}\right)=p(x) \qquad \text{.......... (2-13.5)}$$

Assuming harmonic motion, and since the inertia force is in the same direction of p(x), as shown in Figure 2-13.1 one can write

$$p(x)=\rho\omega^2 y \qquad \text{.......... (2-13.6)}$$

Where ρ is the mass density per unit length of the beam. Then

$$EI\frac{d^4y}{dx4}-\rho\omega^2y=o \qquad \text{....... (2-13.7)}$$

Let

$$B^4=\rho\frac{\omega^2}{EI} \qquad \text{......... (2-13.8)}$$

leads to

$$\frac{d^4y}{dx4}-B^4\ y=o \qquad \text{......... (2-13.9)}$$

The general solution of the above equation which defines the mode shapes takes the form $y = A\cos\beta y + B\sin\beta y + C\sinh\beta y + D\cosh\beta y$ where A, B, C, D are determined from the specified boundary conditions, which also yields the characteristic equation, definding the natural frequency parameters βi. The natural frequencies ω_n's are then found to be

$$\omega_n=\beta_n{}^2\sqrt{\frac{EI}{\rho}}=(\beta_nL)^2\sqrt{\frac{EI}{\rho L^4}} \qquad \text{........ (2-13.10)}$$

2-14 ENERGY METHODS:

The strain energy Uo per unit volume in a body with strains ϵ_{ij} and stresses σ_{ij} which relate the effect of thermal expansion is defined as

$$Uo = \frac{1}{2}\left(\epsilon_{ij} - \alpha T \delta_{ij}\right)\sigma_{ij} \quad \ldots\ldots\ldots\ldots\ldots\ (2\text{-}14.1)$$

using Hooke's Law equations (2-4.2) and (2-5.2) leado to

$$Uo = \frac{1}{2\epsilon}\left(\sigma_{xx}{}^2 + \sigma^2{}_{yy} + \sigma^2{}_{zz}\right) - \frac{\nu}{\epsilon}\left(\sigma_{xx}\,\sigma_{yy} + \sigma_{yy}\,\sigma_{zz} + \sigma_{zz}\,\sigma_{xx}\right)$$
$$+ \frac{1}{2G}\left(\sigma^2{}_{xy} + \sigma^2{}_{yz} + \sigma^2{}_{zx}\right) \quad \ldots\ldots\ldots (2\text{-}14.2)$$

The total energy U is the integral of Uo over the entire volume.

$$U = \iiint_V Uo \ dv \quad \ldots\ldots\ldots\ (2\text{-}14.3)$$

Note that stress strain relations including the effect of thermal expansion can be written using Hooke's law in the form

$$\epsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy}) + \alpha T \quad \ldots\ldots\ (2\text{-}14.4)$$

$$\epsilon_{yy} = \frac{1}{E}\left(\sigma_{yy} - \nu\sigma_{xx}\right) + \alpha T \quad \ldots\ldots\ (2\text{-}14.5)$$

$$\epsilon_{xy} = 2\ G\ \sigma_{xy} \quad \ldots\ldots\ldots\ldots\ (2\text{-}14.6)$$

CHAPTER THREE
GENERAL THEORY

3-1 INTRODUCTION

In this chapter the first and the second law of thermodynamics, Newton's law of motion and Fourier law of heat conduction presented in chapte (2) are taken together to develope the coupled heat conduction equation in a solid medium. Also the main concepts needed to define and to calculate the local and the averaged damping capacity of the structure are introduced. Then the coupled heat conduction equation is solved under adiabatic boundary condition for the rectangular cross section beam-and the magnitude of the local and the averaged damping capacity are obtained in a closed form.

Thermomechanical systems are subjected to the same general conservation laws with regard to both mass and momentum. However the law of conservation of energy contains both mechanical and thermal energy which are related to the change of entropy. Thus a complete description of the evolution of a system requires a knowledge of the entropy production. These laws, taken together, determine the evolution of the system.

To demonstrate the procedure let us consider a solid body occupying a set of rectangular cartesian coordinates. Assume the material is linear elastic and that it is stress free at uniform temperature T_0 when all external forces removed. The stress free state will be denoted as the reference state, and the temperature T_0 as reference temperature.

The displacement u_i of every particle in the instantaneous state from its position in the reference state will be assumed to be small so that the infinitesimal strain components $u_{i,j*}\ u_{j,i}$ are neglected. Thus from equation (2-3.2) the infinitesimal strain tensor is given by

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad \text{................ (3-1.1)}$$

$$= \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] \ , \ i,j = 1,2,3 \qquad \text{..........(3-1.2)}$$

Following are the main equations and definitions which will be used later to develope the coupled heat conduction equation in an elastic solids. From the first law of thermodynamics the energy principle for thermomechanical continuum after substituting equation (2-10.3) into equation (2-4.4) and using equations (2-9.5) after some manipulations yields

where ε : specific internal energy per unit mass

σ_{ij}: stress tensor

ρ: density of the material

$h_{i,i} = q_{i,i}$: heat flux per unit area per unit time

$\frac{d\,\epsilon_{ij}}{dt}$ = rate of deformation tensor given in section 2-7.

$$\frac{d\,\varepsilon}{d\,t} = \frac{1}{\rho}\sigma_{ij}\ \frac{d}{dt}\epsilon_{ij} - \frac{1}{\rho}h_{i,i} \qquad \text{................. (3-1.3)}$$

The thermoelastic Hooke's law for linear elastic solid which relates the stress and strain tensors and including thermal effect is obtained using Hooke's equations including thermal expansion[2] which can be written as

$$\sigma_{ij} = \lambda\,\delta ij\ \epsilon_{kk} + 2\mu\ \epsilon_{ij} - (3\lambda + 2\mu)\alpha\,\sigma_{ij}\,(T - To) \qquad \text{.........(3-1.4)}$$

$$\lambda = \text{Lame's constant} = \frac{E\nu}{(1+\nu)(1-2\nu)} \qquad \text{....... (3-1.5)}$$

$$\mu = G = \frac{E}{2(1+\nu)} \qquad \text{....... (3-1.6)}$$

Also the Fourier law of heat conduction in a solid medium can be written as[1,2]

$$q_i = k_{ij} \cdot T_{,i} \qquad ,i = 1,2,3 \qquad \text{........... (3-1.7)}$$

where

q_i =Heat flux per unit time per unit area
k_{ij} = thermal conductivity tensor

And Newton's law of motion is given by equation (2-10.3)

$$\sigma_{ji,j} + \rho\ Fi = \rho\ \dot{v}i \qquad \text{.............. (3-1.8)}$$

Set $Fi = o$ the above equation simplifies to

$$\sigma_{ij,j} = \rho \frac{\partial^2 ui}{\partial t^2} \qquad \text{............................ (3-1.9)}$$

When entropy is created, mechanical energy is necessarily converted into heat. If we define Δw as dissipation of mechanical energy that is converted into heat during one cycle then

$$\Delta w = \rho To\ Sp \qquad \text{....................... (3-1.10)}$$

Thus the stored elastic energy per unit volume is [13,14]

$$w = \frac{1}{2} C_{ijkL}\, \epsilon_{ij} \epsilon_{kL} \text{........................} (3-1.11)$$

$$\sigma_{ij} - C_{ijkL}\, \epsilon_{kL} \qquad \textbf{Hookes Law}$$

Define w as the maximum stored of elastic energy per sycle for isotropic material, then

$$w = \frac{1}{2}\sigma_{ij} \quad \epsilon_{ij} \qquad \text{............... (3.1.12)}$$

Now we define a local specific damping capacity (SDC) as:

$$\Psi_L = SDC = \frac{\text{Mechanical energy dissipated per cycle}}{\text{Maximum stored elastic energy during the cycle}} \qquad \text{.... (3.1.13)}$$

$$\Psi_L = \frac{\Delta w}{w}$$

Finally the volume - averaged damping Ψ of structure of finite extent may be defined as[3,4]

$$\Psi = \frac{\int_V \Delta w \ dv}{\int_V w \ dv} = \frac{\int_V \Psi_L \ w dv}{\int_V w \ dv} \qquad \text{............................ (3.1.14)}$$

This equation is to be used later in this chapter and in chapter (4) to determine the magnitude of the volume-averaged thermal damping coefficient Ψ after determining the maximum stored elastic energy w, in the total volume of the structure.

3-2 COUPLED HEAT CONDUCTION EQUATION

Based on the last equations, a general form of the coupled heat conduction equation in elastic solids is developed. Define a free energy function F at constant temperature (isothermal process) as

$$F = \varepsilon - Ts \qquad \text{...............(3-2.1)}$$

where ε is the internal energy per unit mass, T temperature, S is the specific entropy, and F is denoted a work function, assumed to be function of strains and temperature

$$F = F(\epsilon_{ij}, T) \qquad \text{............... (3-2.2)}$$

We have also from Clausius - Duhem inequality function [4,5]

$$\rho \dot{\varepsilon} = \sigma_{ij} \dot{\epsilon}_{ij} + \rho T \dot{S} \qquad \text{................ (3-2.3)}$$

Differentiating equations (3-2.1) with respect to time and multipling the resulting equation by ρ; and substituting $\dot{\varepsilon}$ from equation (3-2.3) yields :

$$\rho\dot{F} = \sigma_{ij}\,\dot{\epsilon}_{ij} - \rho\, s\dot{T} \quad \text{.........................} (3-2.4)$$

From second Law of thermodynamics for a reversible process [4,5] $S = F(\epsilon_{ij}, T)$ where S is the entropy , ϵ_{ij} is the strain tensor and T is the temperature:

$$-q_{i,i} = \rho T\dot{s} = \rho T\left(\frac{\partial\ \dot{\epsilon}_{ij}}{\partial\ \epsilon_{ij}} + \frac{\partial\, s}{\partial\, T}\dot{T}\right) \quad \text{...........} (3\text{-}2.5)$$

At constant deformation $\epsilon_{ij} = o$, and the above equation yields

$$-q_{i,i} = \rho\, c_v \dot{T} \quad \text{................} (3\text{-}2.6)$$

where Cv is the specific heat at constant deformation comparing equation (3-2-5) with (3-3.6) one gets

$$Cv = T\frac{\partial\, s}{\partial\, T} \quad \text{............} (3\text{-}2.7)$$

Combining (3-2.3) with (3-2.4) and (3-2.5) and according to the second law of thermodynamics, the Clausius - Duhem inequality [2] can be written as:

$$-q_{i,i} = kT_{,ii} = \rho T\left(\frac{\partial\,\sigma_{ij}}{\partial\, T}\dot{\epsilon}_{ij} + \frac{cv}{T}\dot{T}\right) \quad \text{..............} (3\text{-}2.8)$$

From thermoelastic relations equation (3-1.4) one has

$$\frac{\partial\,\sigma_{ij}}{\partial\, T} = (3\lambda + 2\mu)\alpha\,\delta_{ij}$$

Now substituting equation (3-2.9) into (3-2.8) and after some manipulations one gets the coupled heat conduction equation in the form

$$k\,T_{,ii} = \rho c_v \dot{T} + (3\lambda + 2\mu)\alpha T\, \dot{\epsilon}_{ii} \;..........(3\text{-}2.10)$$

For class of problems under considerations the variation of temperature is very small and ϵ_{ii} may be replaced by total strain ϵ_{kk}. Therefore the coupling term T may be replaced by equillibrium temperature To to obtain one way coupled heat conduction equations:

$$T_{,ii} = \frac{\rho c}{k}\frac{\partial T}{\partial t} + \frac{3\lambda + 2\mu}{k} To \frac{\partial \epsilon_{kk}}{\partial t} ....(3-2.11)$$

$$T_{,ii} = \frac{\rho c}{k}\frac{\partial T}{\partial t} + \frac{E\alpha}{k(1-2\nu)} To \frac{\partial \epsilon_{kk}}{\partial t} \;.................... (3-2.12)$$

This equation when solved determines the temperature distribution inside a contineous elastic solid material. It should be noted that this temperature gradient is due to the variations of stresses and strains-inside the material itself.

3-3 BOUNDARY VALUE PROBLEM 1: FLEXURAL VIBRATIONS OF BEAMS WITH RECTANGULAR CROSS SECTION.

In this section the coupled heat conduction equation (3-2.12) is solved under adiabatic boundary conditions. Consider an isotropic, homogeneous, beam with constant rectangular cross section and height h undergoing harmonic flexural vibrations in the x-y plane. Assume that the centriodal axis occupies the x-axis and the displacement is confined to the y-axis. The curvature of the beam is given by

$$K = K_o \, e^{i\omega t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3\text{-}3.1)$$

where Ko is the initial positive curvature of the beam-and ω is the circular frequency in radians per second. Also assume that the curvature of the beam causes fluctuating temperature across the y-axis of the beam. Now from equation (2-5.3) one gets:

$$\epsilon_{yy} = \epsilon_{zz} = -\nu \;\; \epsilon_{xx} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3-3.2)$$

$$\epsilon_{xx} = -yk = -yk_0 \, e^{iwt} \quad \ldots\ldots\ldots\ldots (3-3.3)$$

$$\epsilon_{kk} = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$$

$$\epsilon_{kk} = -(1-2\nu)k_0 y e^{i\omega t} \quad \ldots\ldots\ldots (3-3.4)$$

The coupled heat conduction equation (3-2.12) can be written as

$$T_{,ii} = \frac{\rho c}{k} \bullet \frac{\partial T}{\partial t} + \frac{E\alpha}{k(1-2\nu)} . To . \frac{\partial \epsilon_{kk}}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots (3\text{-}3.5)$$

Using cartesian coordinates, equation 3-3.5 becomes.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} - \frac{\rho c}{k} \frac{\partial T}{\partial t} = \frac{+E\alpha}{k(1-2\nu)} \frac{\partial \epsilon_{kk}}{\partial t} \quad \ldots\ldots\ldots (3\text{-}3.6)$$

where

$$\epsilon_{kk} = -(1-2\nu)K_0 y e^{i\omega t}$$

$$\frac{\partial \epsilon_{kk}}{\partial t} = -i\omega(1-2\nu)k_0 y \, e^{iwt}$$

Since the displacement is confined to the y-axis equation (3-3.6) reduces to:

$$\frac{\partial^2 T}{\partial y^2} - \frac{\rho c}{k} \frac{\partial T}{\partial t} = -i\omega \frac{E \propto To}{k} k_0 y \;\; e^{iwt} \quad \ldots\ldots\ldots\ldots (3\text{-}3\text{-}7)$$

The beam surfaces are assumed to be adiabatic; therefore the boundary conditions at y = ± (h/2) are:

$$Y = y/h \quad , \quad \Omega = \omega\tau \quad , \quad \overline{N}^* = \frac{N^*}{\Delta T} \quad \text{..................} (3\text{-}3.13)$$

where

$$\tau = \frac{\rho c h^2}{\pi^2 k} \qquad \Delta T = \frac{E h k_0 T_0}{2\rho c}$$

Now substituting these into equation. (3 -3.7) yields:

$$\frac{d^2 \overline{N}^*}{dY^2} - i\Omega\pi^2 N = -2i \;\; \pi^2 \Omega Y \quad \text{...........} (3\text{-}3.14)$$

This is a second order linear differential equation with complex coefficients. Note that $\overline{N}^*$ represents the change of the normalized temperature along a normalized coordinate.

The boundary conditions for this beam are given by

$$\left.\frac{\partial \overline{N}^*}{\partial Y}\right|_{Y=\frac{-h}{2}} = \left.\frac{\partial \overline{N}^*}{\partial Y}\right|_{Y=\frac{h}{2}} = 0.0 \quad \text{...............} (3\text{-}3.15)$$

Equations (3-3.1) through (3-3.15) constitute the general formulation of thermoelastic damping problem of an isotropic, homogeneous constant rectangular cross section beam.

3-4 SOLUTION OF THE DIFFERENTIAL EQUATION

The solution of equation (3-3.14) has two parts: a homogeneous solution and a particular solution. The homogeneous solution to equation (3-3.14) is given by:

$$\bar{N}^*(Y) = A\ e^{\pi(\Omega/2)^{\frac{1}{2}}(1+i)Y} + B\ e^{-\pi(\Omega/2)^{\frac{1}{2}}(1+i)Y} \quad \text{.............. (3-4.1)}$$

where A, B are constants to be determined from the boundary conditions giving by equation (3-3.15). Set $S = \pi(\Omega/2)^{\frac{1}{2}}$ then equation (3-4.1) can be written in the form

$$\bar{N}^*(Y) = A\ e^{S(1+i)Y} + B\ e^{-S(1+i)Y} \quad \text{............ (3-4.2)}$$

The particular solution of equation (3-3.14) takes the form

$$\bar{N}^*(Y) = C_1 . Y + Co \quad \text{........... (3-4.3)}$$

Substituting equation (3-4.3) into equation (3-3.14) one gets $C_1 = 2, Co = 0$, the particular solution takes the form

$$\bar{N}p^* - 2Y \quad \text{..................... (3-4.4)}$$

Therefore the general solution of the boundary value problem can be written as follows:

$$\bar{N}^*(Y) = 2Y + A\ e^{s(1+i)Y} + B\ e^{-s(1+i)Y} \quad \text{............... (3-4.5)}$$

where A, B are constants to be determined from the boundary conditions in equation (3-3.15). Now substituting the boundary conditions from equation (3-3.15) leads to.

$$B = \frac{2}{R_1 + R_2} \qquad \text{.............} \quad (3-4.6)$$

$$A = \frac{B\ R_2 - 2}{R_1}, \qquad \text{..........} \quad (3-4.7)$$

where

$$R_1 = (1+i)s\ e^{\frac{1}{2}(1+i)s} \quad , R_2 = (1+i)s\ e^{\frac{-1}{2}(1+i)s}$$

Therefore the general solution of equation (3-4.14) is given by

$$\bar{N}^*(Y) = 2Y + \frac{2}{R_1 + R_2} \cdot e^{s(1+i)Y} + \frac{BR_2 \quad 2}{R_1} \quad e^{-s(1+i)Y} \quad \text{..........} \quad (3-4.8)$$

Note that the magnitude of change of the normalized temperature N as function of the notmalized coordinate Y is poltted in Figure (3-6.1) for different values of normalized frequencies.

3-5 CALCULATION OF THE LOCAL SPECIFIC DAMPING CAPACITY [SDC].

Local specific damping capacity is defined as Ψ_L

$$\Psi_L = \frac{\Delta w}{w} = \frac{\begin{bmatrix}\text{dissipation of mechanical energy during one cycle} \\ [\text{or part of mechanical energy that is converted into heat}]\end{bmatrix}}{\text{maximum stored strain energy during that cycle}}$$

$$\text{.............} \quad (3-5.1)$$

Using the second law of thermodynamics equation (2-12.4), the rate of entropy created due to irreversible heat conduction in a solid can be calculated as:

$$\frac{\partial sp}{\partial t} = \frac{k}{\rho T^2} T,i \bullet Ti \ldots\ldots\ldots\ldots\ldots(3-5.2)$$

$$T,i = \frac{\partial T}{\partial Y} \Delta T.e^{i\omega t} \frac{\partial T(Y)}{\partial Y} \ldots\ldots\ldots\ldots\ldots(3-5.3)$$

Now differentiating equation (3-4.8) and substituting into equation (3-5.3) leads to:

$$T,i = \frac{\partial T}{\partial Y} = \Delta T \quad e^{i\omega t}\left[2 + \frac{2}{R_1 + R_2}(1+i)s \; e^{s(1+i)Y} - \frac{BR_2 - 2}{R_1}(1+i)s \; e^{-s(1+i)Y}\right]$$

.... (3-5.4)

Now rewritting R1, R2 in terms of trigonometric functions and substituting equation (3-5.4) into equation (3-5.2) yields:

$$\dot{S}p = \frac{k}{\rho}\left[\frac{\left(2 + SG\cos(sY)\cosh(SY) - (\sinh(SY)\sin(SY) + \cosh(SY)\cos(SY)\right)}{\left[2Y + G(\sinh(SY)\mathrm{Cos}(SY) + iG(\cosh(\lambda y)\mathrm{Sin}(\lambda y))\right]^2}\right]$$

.... (3-5.5)

where

$$G = -2\frac{(1-i)}{S}\left[\frac{\sinh\left(\frac{1}{2}S\right)\cos\frac{1}{2}S + i\cosh\left(\frac{1}{2}S\right)\sin\left(\frac{1}{2}S\right)}{\sinh S \cos S + i \cosh S \sin S}\right]$$

$$\Delta w = \rho \, To \; \dot{S}p \qquad \ldots\ldots\ldots\ldots\ldots (3-5.6)$$

From equation (3-1.12) the stored elastic energy per unit volume is given by w

$$W = \frac{1}{2}\sigma ij \quad \in ij \qquad \ldots\ldots\ldots\ldots\; (3-5.7)$$

$$= \frac{1}{2}\left[\in_{xx} \; .\sigma_{xx} + \; \in_{yy} \; . \; \sigma_{yy} + \in_{zz} \; .\sigma_{zz}\right]$$

$$= \frac{E}{2}\left[y^z k^2 + 2v^2 y^2 k^2\right] \quad \ldots\ldots\ldots\; (3-5.8)$$

Noting that k=Ko $e^{i\omega t}$ and substituting into equation (3-5.8) leads to

$$W = \frac{Eh^2}{2} Ko^2\left(1 + 2v^2\right)Y^2 e^{2iwt} \quad \text{.......................(3-5.9)}$$

Substituting these values of S_p and W, ΔW into equation (3-5.1) and noting that

$$\Psi o = \frac{2\pi \propto^2 E_R \ To^2}{\rho \ c} \quad \text{......... (3-5.10)}$$

one obtains:

$$\Psi L = \frac{\Psi o}{{}^2S^2Y^2}\left\{1 - \frac{1}{\cosh(2S) - \cos(2S)}\right.$$

$$x\left[2\cos\left[S\left(Y - \frac{1}{2}\right)\right]\cosh\left[S\left(Y + \frac{3}{2}\right)\right]\right.$$

$$+ 2\cos\left[S\left(Y + \frac{1}{2}\right)\right]\cosh\left[S\left(Y - \frac{3}{2}\right)\right]$$

$$- 2\ \mathrm{Cos}\left[S\left(Y + \frac{3}{2}\right)\right]\mathrm{Cosh}\left[S\left(Y + \frac{1}{2}\right)\right]$$

$$2\mathrm{Cos}\left[S\left(Y - \frac{3}{2}\right)\right]\mathrm{Cosh}\left[S\left(Y + \frac{1}{2}\right)\right]$$

$$+ 2\cos(S)\mathrm{Cosh}(2SY)$$

$$- 2\mathrm{Cos}(2SY)\mathrm{Cos}(S)$$

$$+ \cos\left[2S\left(Y + \frac{1}{2}\right) + \cos\left[2S\left(Y - \frac{1}{2}\right)\right]\right.$$

$$\left. - \cosh\left[2S\left(Y + \frac{1}{2}\right)\right] - \cosh\left[2S\left(Y - \frac{1}{2}\right)\right]\right\} \text{.........(3-5.11)}$$

From equation (3-1.14) the volume-averaged damping is defined by:

$$\Psi = \frac{\int_V \Delta w \ dv}{\int_V w dv} = \frac{\int_V \Psi L w dv}{\int_V w dv} \quad \text{................ (3-5.12)}$$

Where w is given by equation (3-5.9)

$$W = \frac{Eh^2Ko^2}{2}\left(1+2v^2\right)Y^2 \ e^{2i\omega t} \qquad .... (3\text{-}5.13)$$

Substituting the value of W from equation (3-5.13) into equation (3-5.12) leads to

$$\Psi' = \frac{\int_{-\frac{1}{2}}^{+\frac{1}{2}} \Psi L . Y^2 \ dY}{\int_{-\frac{1}{2}}^{\frac{1}{2}} Y^2 \ dY}$$

Substituting the value of ΨL from equation (3.5.11) and integrate with respect to Y one obtains

$$\Psi' = \frac{6\Psi o}{S^2}\left\{1 - \frac{1}{S}\left[\frac{\sinh(2S) - \text{Sin}(2S)2 - \text{Cos}(S)\sinh(S) + 2\sin(S)\cosh(S)}{\cosh(2S) - \cos(2S)}\right]\right\}$$

.......(3-5.14)

3-6 BOUNDARY VALUE PROBLEM 1: NUMERICAL RESULTS

Now going back to equation (3-4.8), and assume No and ϕ to be respectively the magnitude and the phase of the normalized tempetature $\bar{N}^*$, one can write

$$\bar{N}^* = No \ \ e^{i\phi} \qquad .... (3\text{-}6.1)$$

From equation (3-4.8) and after some manipulations the values of No and ϕ becomes:

$$No = \left[\left[2Y - \overline{\frac{2(M+N)}{s(c^2+D^2)}}\right]^2 + \left[\overline{\frac{M-N}{C^2+D^2}}\right]^2 \right]^{\frac{1}{2}} \qquad .... (3\text{-}6.2)$$

$$\phi = \tan^{-1}\left[\frac{\left[\frac{M-N}{C^2+D^2}\right]}{\left[2Y - \frac{2(M+N)}{S(C^2+D^2)}\right]}\right] \qquad \text{.................. (3-6.3)}$$

From the above equation one can see that the volume averaged thermal damping coefficient is only function of the normalized frequency for constant values of Ψo, where

$$M = c[a.E - b.F] + D[b.E + a.F] \text{.............}(3-6.4)$$
$$N = c[b.E + a.F] + D[b.F - a.E] \text{..............}(3-6.5)$$

$$a = \sinh\left[\frac{1}{2}S\right]\cos\left[\frac{1}{2}S\right] \qquad b = \cosh\left[\frac{1}{2}S\right]\sin\left[\frac{1}{2}S\right]$$
$$C = \sinh[S]\cos[S] \qquad D = \cosh[S]\sin[S]$$
$$E = \sinh(SY)\cos(sy) \qquad F = \cosh[SY].\sin[SY]$$

.... (3-6.6)

Now two cases will be considered:

1- When the frequency is very small, the heat generated in the beam has sufficient time to conduct from regions of elevated temperature to regions of lowered temperature. As a result the temperature remains at the reference temperature, hence, the magnitude of No is very small and appraches zero. For example No≈o at $\Omega = 10^{-3}$ (which is correspondes to isothermal boundary conditions).

2- On the other hand, when the frequency is very large the heat has no time to conduct from regions of elevated temperature to regions of lowered temperature; therefore, the temperature corresponds to adiabatic boundary conditions. From the solution of the differential equation(3-4.8)we see that as the frequency approaches infinity $\Omega \to \infty$

$$\bar{N}^{*} = 2Y$$

This solution is evidenced by a nearly linear variation of No with Y at $\Omega = 10^3$. Figure (3-6.1) shows spatial dependence of the magnitude of normalized temperature for fixed frequencies and from this figure one can see that $\bar{N}^*$ slightly changes with Y for smaller values of normalized frequency, and the variation of $\bar{N}^*$ with Y becomes linear for larger values of Y. Figure (3-6.2) shows the phase of ϕ as function of Y and from this figure one can say that for all small values of Ω $\bar{N}^*$ is independent of Y. Since $\bar{N}^*$ is an odd Function of Y, then:

$$\phi(Y) = \phi(-Y) + \pi$$

Accordingly, we have considered $o \le Y \le \frac{1}{2}$ only. Now let us consider the case when $\Omega \to 0.0$:

$$\phi = \tan^{-1}\left[\frac{\left[\dfrac{M-N}{C^2+D^2}\right]}{\left[2Y - \dfrac{2(M+N)}{S(C^2+D^2)}\right]}\right]$$

Substituting for the valuse of a, b, c, D, E, F, M, N we get:

$a = \sinh(o)\cos(o) = 0.0$, $b = \cosh(o)\sin(o) = 0$

$C = \sinh(o)\cos(o) = 0$, $D = \cosh(o)\sin(o) = 0$

$E = \sinh(o)\cos(o) = 0$, $F = \cosh(o)\sin(o) = 0$

$M = 0$ $N = 0$

Substituing in ϕ we get:

$$\phi = \tan^{-1}\left[\frac{0.0}{0.0}\right] = \frac{\pi}{2}$$

This means that the temperature leads the curvature by angle $\left(\frac{\pi}{2}\right)$, and essentially is independent of Y. Alo as $\Omega \to \infty$ the tempardture is in phase with the curvature, as expected. Figure (3-6.3) shows the frequency dependence of $\bar{N}^*$ for fixed values of Y. Since No is an even function of Y

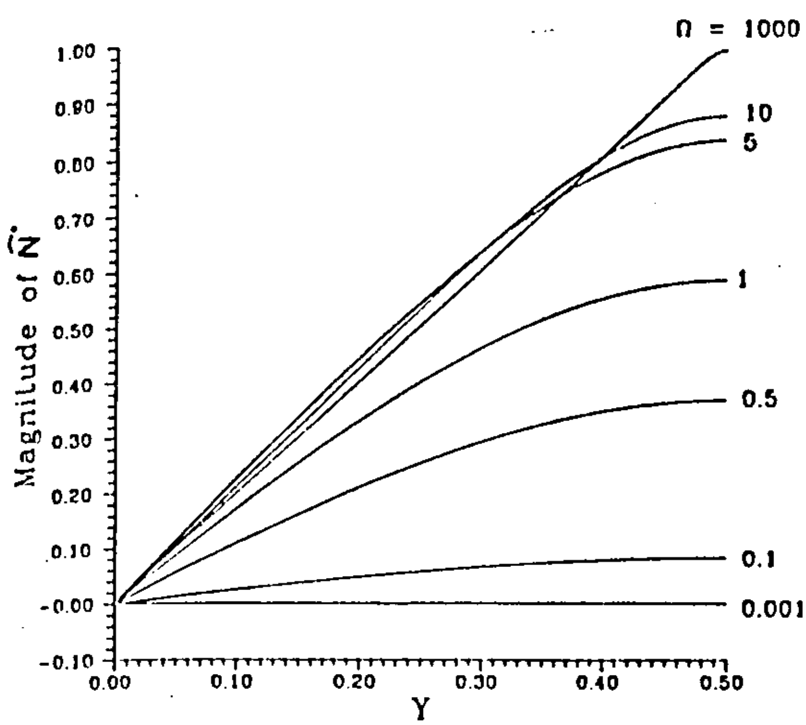


Figure [3-6.1] Spatial dependence of the magnitude of the normalized temperature for fixed frequencies [3].

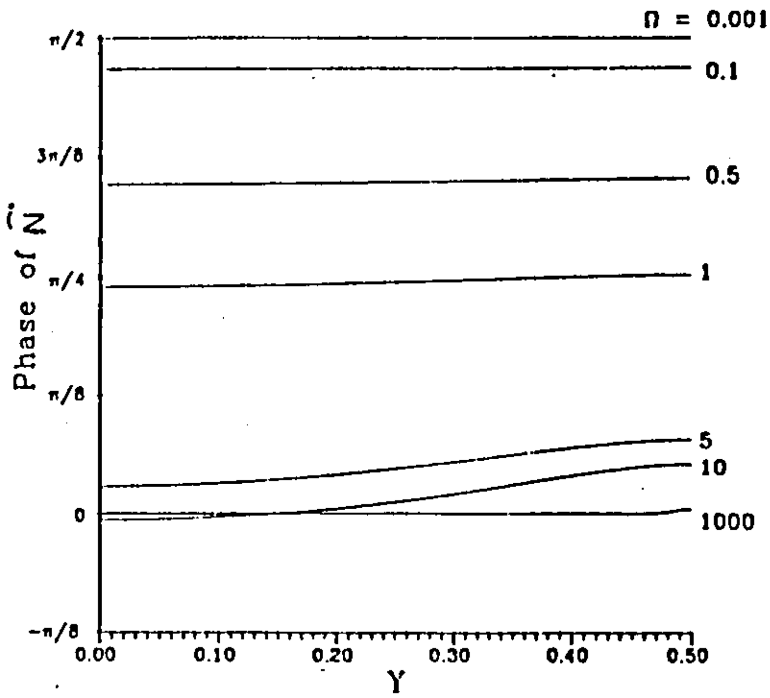


Figure [3-6.2] Spatial dependence of the phase of the normalized temperature for fixed frequencies[3]

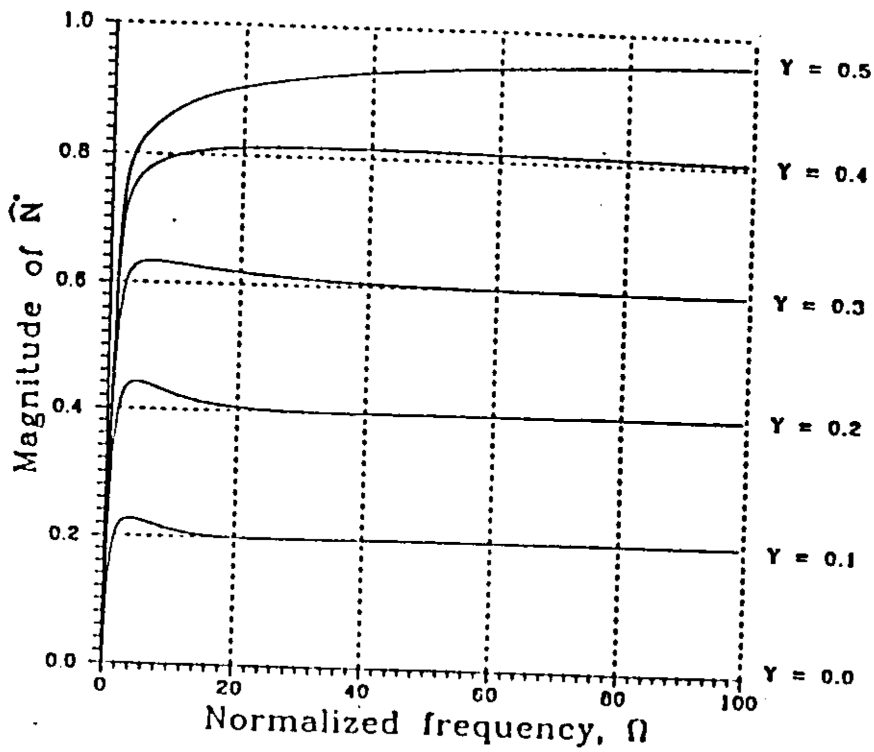


Figure [3-6.3] Frequency dependence of the magnitude of the normalized temperature for fixed Y [3].

3-7 VOLUME AVERAGED DAMPING COEFFICIENT

Next we consider the Volume averaged damping Ψ

$$\Psi = \frac{\sigma \Psi o}{S^2}\left\{1-\frac{1}{S}\left[\frac{\sinh(2S)-\sin(2S)-2\cos(S)\sinh(S)+2\sin(S)\cosh(S)}{\cosh(2S)-\cos(2S)}\right]\right\}$$

.........(3-7.1)

where

$$S = \pi(\Omega/2)^{\frac{1}{2}} \qquad , \Omega = \omega\,\tau$$

$$\tau = \frac{\rho\, ch^2}{\pi^2 k^2} \qquad , \Psi o = \frac{2\rho\alpha^2 E T_0}{\rho\, c}$$

Figure(3-7.1)shows the frequency dependence of normilized damping $\frac{\Psi}{\Psi o}$.

It is important to note that this curve is a universal curve for all beams of rectangular cross section in flexure; each beam is classified by two of its thermoelastic properties:

1. The characteristic damping, Ψo, which is a material property
2. The characteristic time, τ, which is a combination of material properties and structural property (h). It can be easly shown from Figure (3-7.1) that there is a maximum value of normalized damping $\left(\frac{\Psi}{\Psi o}\right)$ corresponding to certain frequency Ω (say Ω=1.0). i.e $\left.\frac{\Psi}{\Psi_0}\right|_{max} = 0.5$.

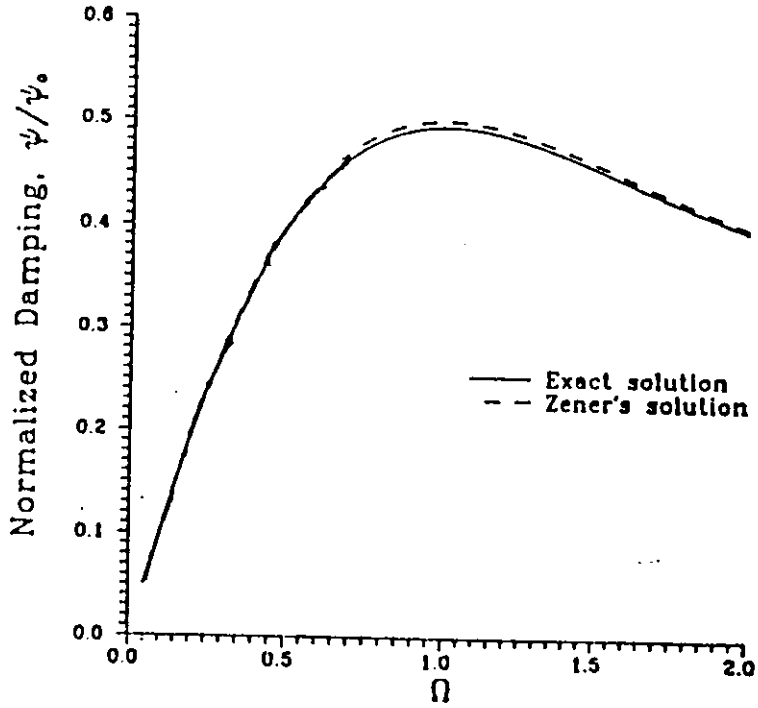


Figure (3-7.1) A comparison of the present exact solution with the solution of zener

CHAPTER FOUR

THERMAL DAMPING COEFFICIENTS IN ELASTIC PERFECTLY PLASTIC MEDIUM

4-1 INTRODUCTION:

In this chapter the magnitude of thermal damping coefficients in elastic perfectly plastic medium is evaluated based on the basic concepts of thermodynamics and plasticity. The solid material is assumed perfectly plastic and the relation between stress and strain obey's the total deformation theory.

Although it is important to say that temperature have a definite influence upon plastic behavior of real materials, it is customary in plasticity to assume isothermal conditions and consider temperature as a parameter. One of the most used model in analyzing plastic behavior of real materials is that one shown in Figure (4-1.1) below where σ_y is yield stress.

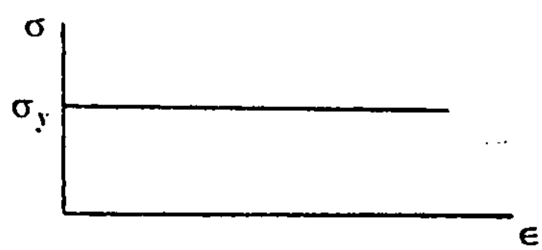


Figure 4-1.1

The Tresca yield condition (Maximum Shear Theory)[2], says that yielding occurs when maximum shear stress reaches the prescribed value (constant value) C_Y. When referred to the yield stress in simple tension this constant C_Y is equal to $\frac{1}{2}\sigma_y$, where σ_y is the yield stress, i.e

$$\sigma_I - \sigma_{III} = \sigma_Y \;,\; \frac{1}{2}(\sigma_I - \sigma_{III}) = C_Y \quad .... (4\text{-}1.1)$$

where σ_I, σ_{III}, are the principal stresses obtained from Mohor's Circles.

4-2 BASIC EQUATIONS WHICH COVER THE MOTION OF ELASTIC PLASTIC MEDIUM

In this section the basic equations which cover the motion of elastic plastic medium are summarized. These equations are used later on to develope the coupled heat conduction equation in elastic - plastic medium.

For a contineous medium undergoing small strains and rotations the equations of motion are using (Newton's Law):

$$\sigma_{ij,i} = \rho\ \dot{\upsilon}_i \qquad \text{.................. (4-2.1)}$$

where σ_{ij} is the stress tensor, ρ is the mass density per unit volume and υi is the velocity vector. Using equation (3-1.1) the strain displacement equations may be written as

$$\dot{\epsilon}_{ij} = \frac{1}{2}\left(\upsilon_{i,j} + \upsilon_{j,i}\right) \text{.......................................} (4.2.2)$$

where σij and ϵ_{ij} are respectively, the components of stress and strain, υi are particle velocity components, and ρ is the constant mass density. In an elastic state the medium is assumed to be linear, homogeneous and isotropic. If plastic response is governed by the Von-Mises Criterion and isotropic work hardening, the complete stress-strain equation becomes [2]

$$\dot{\epsilon}_{ij} = Q_{ijkl}\ \dot{\sigma}_{kl} \qquad \text{..................... (4-2.3)}$$

where

$$Q_{ijk} = \overset{-1}{C_{ijkl}} + G \bullet f_{ij} \bullet f_{kL} \text{..................} (4-2.4)$$

$$\overset{-1}{C_{ijkL}} = \frac{1+\nu}{E}\ \delta_{ik}\ \delta_{iL} - \frac{\nu}{E}\ \delta_{ij}\ \delta_{kL} \text{.......} (4-2.5)$$

where ν is poisson's ratio, δ_{ik} is the kronecker delta, and E is Young's modulas, and

$$f_{ij}=\frac{\partial f}{\partial \sigma_{ij}} \qquad ....(4\text{-}2.6)$$

where f is the loading function given by [9, 11]

$$f=\frac{1}{2}S_{ij}, \quad S_{ij}=K^{2} \qquad ....(4-2.7)$$

$$S_{ij}=s_{ij}-\frac{1}{3}s_{kk}\ \delta_{ij} \qquad ....(4-2.8)$$

and K is the yield stress in pure shear. Finally G is a scalar function of K and controls the plasticity of the material. Inverting equation (4-2.3) yields:

$$\dot{\sigma}_{ij}=P_{ijkL}\ \dot{\epsilon}_{KL} \qquad ....(4-2.9)$$

where

$$P_{ijkl}=C_{ijkl}-\mu|\delta_{ik}\ \delta_{jL}+\delta_{ik}\delta_{iL}| \qquad ....(4-2.10)$$

$$C_{ijkL}=\lambda\delta_{ij}\delta_{kL}+\mu\left(\delta_{ik}\ \delta_{jL}+\delta_{ik}\ S_{iL}\right) \qquad ....(4-2.11)$$

where λ and μ are lame constants, and $h=\dfrac{4\mu\ GK^{2}}{1+4\ \mu\ GK^{2}}$ ..(4-2.12)

The case h = 0 corresponds to an elastic state and h = 1 to perfectly plastic state. Substitution of equation (4-2.2) into (4-2.9) leads to:

$$\dot{\sigma}_{ij}=P_{ijkL}\ \upsilon_{k,L} \qquad ....\ (4\text{-}2.13)$$

Equations (4-2.1) and (4-2.13) which governs the motion of the elastic - plastic medium, are a system of nine - first order linear parial differential equations with nine unknowns σ_{ij}, v_i .

4-3 STRESS STRAIN RELATIONS AND ENERGY EQUATION FOR ISOTROPIC LINEAR ELASTOPLASTIC SOLIDS.

In this section the stress strain relation and energy equation for an isotropic linear elastoplastic solid are derived from thermodynamic principles. Some necessary concepts will be introduced first with reference to the one dimensional model and will then be extended to a general, elastroplastic solid and to three - dimensional states of stress.

First it assumed that the assumption of small displacements and small displacement gradients is valid, therefore one can decompose the strain tensor into elastic and plastic components. Now using the assumed model of Figure. (4-3.1) let e be the total strain of the entire model and let $\overset{n}{e}$, n = 1,2,3 , be the total strain of legs 1, 2, 3.[6,17].

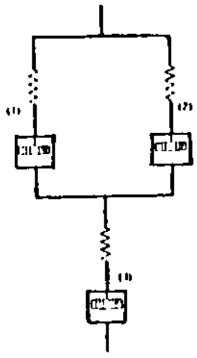


Figure (4-3.1)

Then from purely kinematical considerations, one can write:

$$\overset{}{e} = \overset{1}{e} + \overset{3}{e}, \overset{1}{e} = \overset{2}{e} \text{ ....(4-3.1)}$$

Also assume e_e^n denote the elastic strain in each leg and e_p^n denote the plastic strain in each leg , then

$$\overset{n}{e} = \overset{n}{e}_e + \overset{n}{e}_p, \quad n = 1,2,3 \text{.............(4 – 3.2)}$$

$$\overset{1}{e} = \overset{1}{e}_e + \overset{1}{e}_p, \overset{2}{e} = \overset{2}{e}_e + \overset{2}{e}_p, \overset{3}{e} = \overset{3}{e}_e + \overset{3}{e}_p$$

devolope the coupled heat conduction equation for elastic plastic medium, which yield:

$$\rho c \dot{T} = \dot{\sigma}_{ij} \dot{\epsilon}^{p}_{ij} - \alpha T \dot{\sigma}_{kk} - q_{i,i} + \rho R \text{ .... (4-3.7)}$$

where ρ is the mass density per unit volume , c is the specific heat per unit mass, α is the coefficient of thermal expansion, T is the absolute equilibrium temperature, R is the heat flux supply per unit volume, $\dot{\epsilon}^{p}_{ij}$ is the rate of change of the plastic strain tensor per unit time, and $\dot{\sigma}_{kk}$ is the rate of change of stress tensor per unit time.

Now for this class of problems i.e elastic perfectly plastic medium one assumes that the rate of change of plastic strain tensor with respect to time is negligible, therefore:

$$\dot{\epsilon}^{p}_{ij} = 0.0 , \qquad q_{i,i} = -kT_{,ii} \text{ ....(4-3.8)}$$

where K is thermal conductivity of the medium. One may assume that there is no heat supplied to the system i.e

$$\rho R = 0.0 \text{ .... (4-3.9)}$$

Now rewriting equation (4-3.7) using equations (4-3.8) and (4-3.9) leads to

$$\rho C \dot{T} = -\alpha T_0 \dot{\sigma}_{kk} + kT_{,ii} \text{ ....(4-3.10)}$$

or

$$T_{,ii} = \frac{\rho c}{k} \dot{T} + \frac{\alpha T_0}{k} \dot{\sigma}_{kk} \text{ .... (4-3.11)}$$

Equation (4-3.9) is the coupled heat conduction equation for elasticplastic medium which contains both thermal and mechanical term.

One may also assume that the plastic deformation process is irreversible process, [7,10] and the entropy created due to this process can be calculated from the second law of thermodynamics [2] as follows:

$$\frac{\partial Sp}{\partial t} = \frac{k}{\rho T^2} T_{,i} \; T_{,i} \quad \text{...................(4-3.12)}$$

where Sp is the entropy produced per unit mass. When the entropy is created part of mechanical energy is converted into heat, according to

$$\Delta W = \rho \, To \, \dot{S}p \quad \text{.................................. (4-3.13)}$$

The stored energy can be calculated from [3]

$$W = \frac{1}{2} \sigma_{ij} \; \epsilon_{ij} \quad \text{..........................(4-3.14)}$$

Consider a thermoelastic perfectly plastic solid undergoing a steady time - harmonic deformation. Assume that ΔW defines the dissipation of mechanical energy per cycle, and W identifies the maximum stored strain energy during that cycle. Define a local specific damping capacity (SDC) as:

$$\Psi_L = \frac{\Delta W}{W} \quad \text{..................(4-3.15)}$$

Also as before define the volume - averaged damping as:

$$\Psi = \frac{\int \Delta W . dv}{\int W . dv} = \frac{\int \Psi_L W . dv}{\int W . dv} \quad \text{................(4-3.16)}$$

Equations (4-3.6) through (4-3.16) constitute the general theory of thermoplastic damping. In the following we will solve heat conduction equation for adaibatic boundary conditions.

(4-4) BOUNDARY VALUE PROBLEMS :

FLEXURAL VIBRATIONS OF RECTANGULAR CROSS SECTION BEAMS IN PLASTIC MEDIUM

In this section the coupled heat conduction equation is used to develope the general theory of thermal damping in elastic-perfectly plastic solids. Consider an isotropic, homogeneous, thermo elasto plastic, beam with constant rectangular cross section and height h undergoing time harmonic flexural vibrations in the x-y plane.

Assume that the change of original dimensions of the beam is very small, therefore the beam cross section is still rectangular after deformation. Also assume that the material of the beam is elastic perfectly plastic that the centeriodal axis occupies x-axis and the displacement is confined to the y-axis. Let the curvature of the beam be given by

$$K=Ko\ e^{i\omega t} \quad \text{......................(4-4.1)}$$

where Ko is the initial curvature of the beam, and ω is the circular frequency in radians per second.

The total stress can be written using total deformation theory as [2]

$$\sigma_{kk} = -Ek_0 y\ e^{i\omega t} \quad \text{.................... (4-4.2)}$$

where v is the poisson's ratio, and E is the Young's modulas. Now

$$\dot{\sigma}_{kk} = -\ EK_0 y.\ i\omega\ \ e^{i\omega t} \text{............}(4-4.3)$$

Substituting equation (4-4.3) in to (4-3.9) yields

$$\frac{\partial^2 T}{\partial y^2} - \frac{\rho\, c}{k}\frac{\partial T}{\partial t} = -\frac{\alpha ToEi\omega\, k_0}{k}\ \ e^{i\omega t} \text{...........}(4-4.4)$$

The beam surfaces are assumed to be adiabatic, i.e :

$$\frac{\partial T}{\partial y}\left(\frac{-h}{2},t\right)=\frac{\partial T}{\partial y}\left(\frac{h}{2},t\right)=0.0 \quad \text{..........(4-4.5)}$$

Next introduce a complex temperature which takes the form

$$T=\overset{*}{N}(y)\ e^{i\omega t} \quad \text{.....................................(4-4.6)}$$

where $\overset{*}{N}(y)$ is the unknown spatial variation of the temperature in y-direction a cross the beam. Using Thomson effect the decrease in temperature can be calculated as:

$$\Delta T=-\frac{\alpha}{\rho c}\sigma.To \quad \text{................ (4-4.7)}$$

Define the normalized temperature $\bar{N}^{*}$ as follows :

$$\bar{N}^{*}=\frac{\overset{*}{N}}{\Delta T} \quad \text{....................................... (4 -4. 8)}$$

where Δ T is the calculated temperature variation from Thomson effect i.e :

$$\Delta T=\frac{\alpha\, h\, E\, T_0 K_0}{2\rho c} \quad \text{.................................. (4 - 4.9)}$$

In the same way define a normalized coordinate as:

$$Y=\frac{y}{h} \quad \text{.................... (4 - 4.10)}$$

Also define a normalized frequency as:

$$\Omega = \omega\tau \qquad \text{.................(4-4.11)}$$

where τ is the characteristic time of the bean defined as:

$$\tau = \frac{\rho\, ch^2}{\pi^2 k} \qquad \text{...................(4-4.12)}$$

Substituting equation (4 - 4.6) through (4- 4.12) into equation (4- 4.4) yields:

$$\frac{d^2 \bar{N}^*(Y)}{dY^2} - i\,\Omega\,\pi^2\ \bar{N}^*(Y) = -2i\pi^2.\Omega\, Y \text{....(4-4.13)}$$

One can see that this equation which describes the temperature variation along y-axis is similiar to that in elastic range. The only difference is the magnitude of the normalized temperature which differes by ΔT.

$$(\Delta T)\text{plastic} = \frac{(\Delta T)\text{elastic}}{(1-2\nu)} \qquad \text{.................. (4-4.14)}$$

(4 - 5) : SOLUTION OF BOUNDARY VALUE PROBLEMS:

In this the section coupled heat conduction equation (4-4.13) is solved and the magnitude of the thermal damping coefficient is calculated. Following the same procedure for solving boundary value problem in the elastic range the following is obtaned:

$$\bar{N}^*(Y) = \bar{N}^*(Y)\Big|_{\text{Homogeneous}} + \bar{N}^*(Y)\Big|_{\text{particular}}$$

$$= 2Y + C_1 e^{(1+i)H} + C_2 e^{-(1+i)H} \text{.........(4-5.1)}$$

where C_1, C_2 are constants to be determined from the adiabatic boundary conditions, and H is a constant defined according to the roots of the homogenous part of the differential equation i.e:

$$\frac{d\bar{N}^{*}(Y)}{dY^{2}} - i\Omega\pi^{2}\bar{N}^{*}(Y) = 0.0 \quad \text{.....................} (4\text{-}5.2)$$

Thus by comparasion with the standard form of differential equations with complex coefficient one obtains:

$$H = \left[\frac{\Omega\pi^{2}}{2}\right]^{\frac{1}{2}} = \pi\left(\frac{\Omega}{2}\right)^{\frac{1}{2}} \quad \text{.................} (4\text{-}5.3)$$

Now write equation (4-5.1) in the form

$$\bar{N}^{*}(Y) = 2Y + C_{11}\, i\sinh(HY)\cos(HY) + C_{12} i \cosh(HY)\sin(HY)$$

$$\text{.......} (4\text{-}5.4)$$

where C_{11}, and C_{12} are constant's to be determined from boundary conditions. From the adiabatic boundary conditions one has:

$$\frac{\partial \bar{N}^{*}}{\partial Y}\left(\frac{-1}{2}, t\right) = \frac{\partial \bar{N}^{*}}{\partial Y}\left(\frac{1}{2}\right) = 0.0 \quad \text{.......................} (4\text{-}5.5)$$

From equation (4-5.5) one obtains

$$\left.\frac{\partial \bar{N}^{*}}{\partial Y}\right|_{Y=\left(\frac{-1}{2}\right)} = 2 + C_{11} i\left[H\cosh(-\tfrac{1}{2}HY)\cos(-\tfrac{1}{2}HY) - H\sinh(-\tfrac{1}{2}HY)\sin(-\tfrac{1}{2}HY)\right]$$

$$+ C_{12} i\left[H\,\mathrm{Sinh}(\tfrac{1}{2}HY)\,\mathrm{Sin}\,(-\tfrac{1}{2}HY) + H\cosh(-\tfrac{1}{2}HY)\cos(-\tfrac{1}{2}HY)\right] \text{.........} (4-5.6)$$

$$\left.\frac{\partial N(Y)}{\partial Y}\right|_{Y=\frac{+1}{2}} = 2 + C_{11} iH\left[\cosh\left(\tfrac{1}{2}H\right)\cos\left(\tfrac{1}{2}H\right) - \sinh\left(\tfrac{1}{2}H\right)\sin\left(\tfrac{1}{2}\right)H\right]$$

$$+ C_{12} iH\left[\sinh\left(\tfrac{1}{2}H\right)\sin\left(\tfrac{1}{2}H\right) + \cosh\left(\tfrac{1}{2}H\right)\cos\left(\tfrac{1}{2}H\right)\right] = 0.0 \text{............} (4-5.7)$$

Solving equation's (4-5.6) and (4-5.7) for C_{11} and C_{12} and substituting into equation 4-5.5 leads to :

$$\overset{-*}{N}(Y)=2Y-2\frac{(1-i)}{H}\left[\frac{\sinh\left(\frac{1}{2}H\right)\cos\left(\frac{1}{2}H\right)+i\cosh\left(\frac{1}{2}H\right)\sin\left(\frac{1}{2}H\right)}{\sinh(H)\cos(H)+i\cosh(H)\sin(H)}\right]$$

$$*\left[\sinh(HY)\cos(HY)+i\cosh(HY)\sin(HY)\right]$$

.........(4-5.8)

Now calculate the entropy created according to the relation

$$\frac{\partial\, sp}{\partial\, t}=\dot{S}p=\frac{k}{\rho T^2}T,i\;\;T,i$$

$$\dot{S}p=\frac{k}{\rho\left[\overset{-*}{N}(Y)\right]^2}\cdot\frac{1}{h^2}\cdot\left[\frac{\partial\,\overset{*}{N}(Y)}{\partial\, Y}\right]^2 \quad ....(4-5.9)$$

Assume B as

$$B=\frac{-2(1-i)}{H}\left[\frac{\sinh\left(\frac{1}{2}H\right)\cos\left(\frac{1}{2}H\right)+i\cosh\left(\frac{1}{2}H\right)\sin\left(\frac{1}{2}H\right)}{\sinh(H)\cos(H)+i\cosh(H)\sin(H)}\right]....(4-5.10)$$

Combining the above relations , yields

$$\overset{-*}{N}(Y)=\frac{\overset{*}{N}(Y)}{\Delta T}=2Y+B\left[\sinh(HY)\cos(HY)+i\cosh(HY)\sin(HY)\right]....(4-5.11)$$

Or

$$\overset{*}{N}(y)=\Delta T\left\{2Y+B\left[\sinh(HY)\cos(HY)+i\cosh(HY)\sin(HY)\right]\right\}....(4-5.12)$$

$$\frac{\partial \overset{*}{N}(Y)}{\partial Y}=\Delta T\left\{\frac{2+HB\left[\cosh(HY)\cos(HY)-\sinh(HY)\sin(HY)\right]}{iHB\left[\sinh(HY)\sin(HY)+\sinh(HY)\cos(HY)\right]}\right\}...(4-5.13)$$

Squaring equation (4-5.13) and also squaring equation (4-5.11)and substituting into (4-5.9) one gets the local specific damping capacity in the elastic perfectly plastic solid as:

$$\Psi_L = \frac{\Psi_0}{2H^2Y^2}\left\{1 - \frac{1}{\cosh(2H) - \cos(2H)}\right.$$
$$\times\left[2\cos\left[H\left(Y - \frac{1}{2}\right)\right]\cosh\left[H\left(Y + \frac{3}{2}\right)\right]\right.$$
$$+ 2\cos\left[H\left(Y + \frac{1}{2}\right)\right]\cosh\left[H\left(Y - \frac{3}{2}\right)\right]$$
$$- 2\cos\left[H\left(Y + \frac{3}{2}\right)\right]\cosh\left[H\left(Y - \frac{1}{2}\right)\right]$$
$$+ 2\cos\left[H\left(Y - \frac{3}{2}\right)\right]\cosh\left[H\left(Y + \frac{1}{2}\right)\right]$$
$$+ 2\cos(H)\cosh(2HY)$$
$$- 2\cos(2HY)\cosh(H)$$

$$+ \cos\left[2H\left(Y + \frac{1}{2}\right)\right] + \cos\left[2H\left(Y - \frac{1}{2}\right)\right]$$
$$\left.\left. - \cosh\left[2H\left(Y + \frac{1}{2}\right)\right] - \cosh\left[2H\left(Y - \frac{1}{2}\right)\right]\right]\right\} \text{....}(4-5.14)$$

Integrating over Y from $\left(\frac{-1}{2} \text{ to } \frac{+1}{2}\right)$ one obtains the volume - averaged damping Ψ as

$$\Psi = \frac{6\Psi_0}{H^2}\left\{1 - \frac{1}{H}\left[\frac{\sinh(2H) - \sin(2H) - 2\cos(H)\sinh(H) + 2\sin(H)\cosh(H)}{\cosh(2H) - \cos(2H)}\right]\right. \text{...........}(4\text{-}5.15)$$

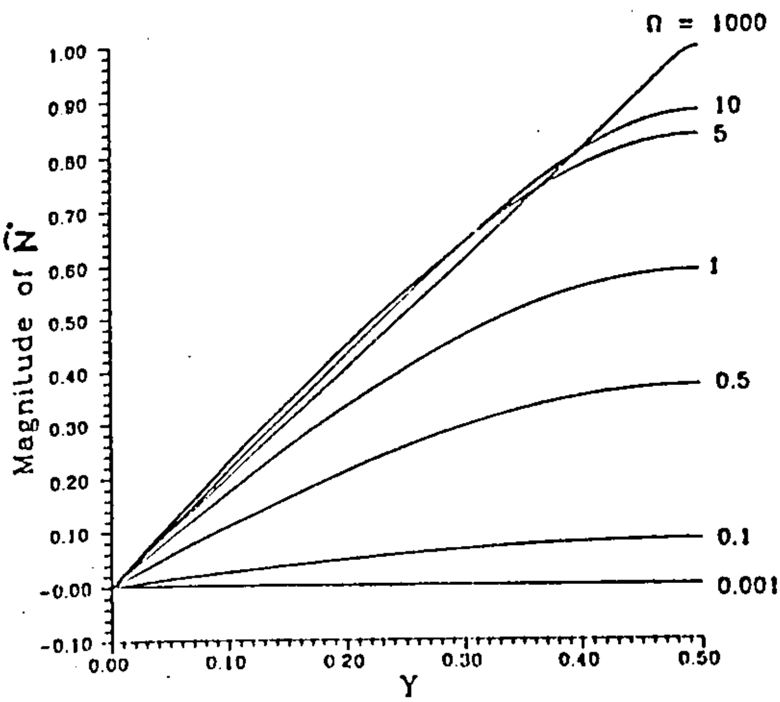


Figure [4-5.1] Spatial dependence of the magnitude of the normalized temperature for fixed frequencies [3].

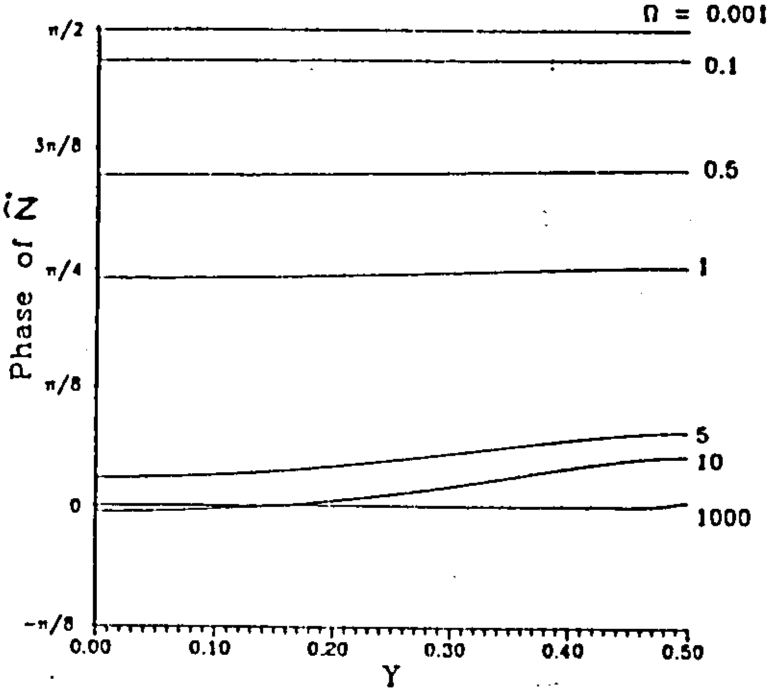


Figure [4-5.2] Spatial dependence of the phase of the normalized temperature for fixed frequencies[3]

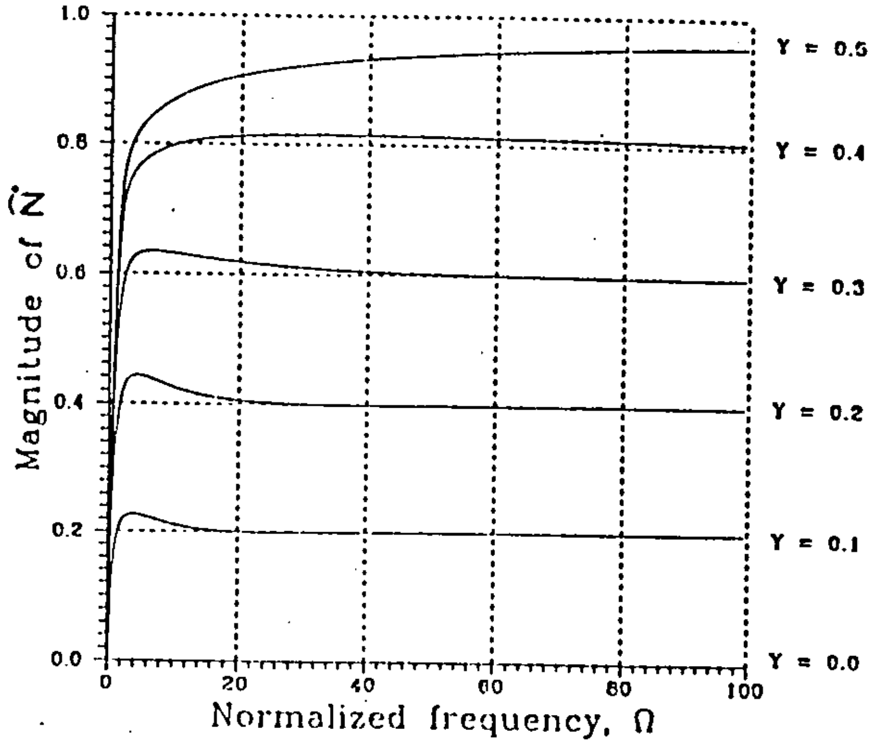


Figure [4-5.3] Frequency dependence of the magnitude of the normalized temperature for fixed Y [3].

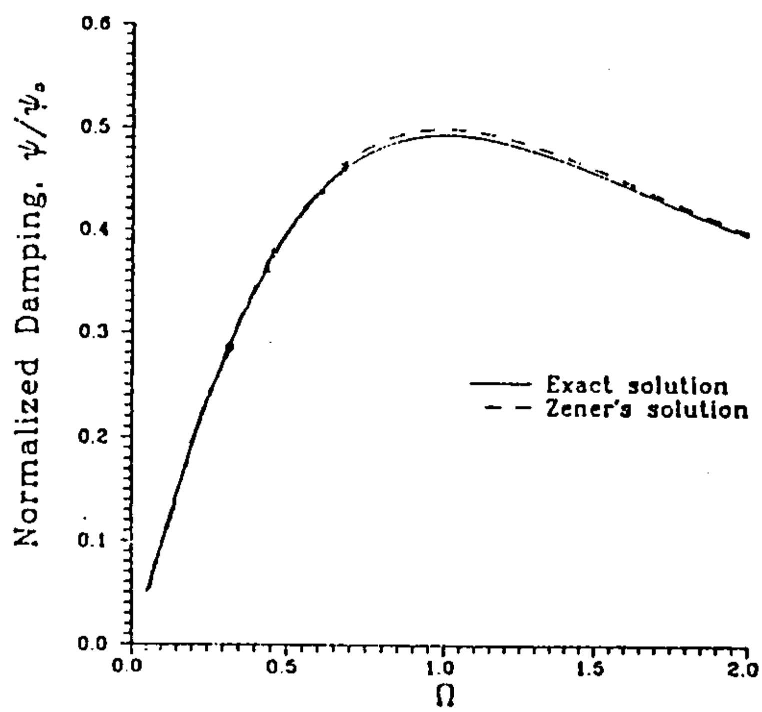


Figure (4-5.4) A comparison of the present exact solution with the solution of zener

CHAPTER FIVE
EIGEN SOLUTIONS FOR COUPLED THERMAL ELASTO -PLASTIC VIBRATIONS AND DAMPING FOR RECTANGULAR CROSS SECTION BEAMS.

5-1 INTRODUCTION

In this chapter the general formulation of the vibration boundary value problem was presented and the governing equations for the free-vibration boundary value problem of rectangular cross section beam under general mechanical boundary conditions and thermal boundary conditions that follows Newton's surface heat exchange law are presented.

Let us Consider a rectangular cross-section beam of thickness 2a and length L. Let t be the time, and let x be the axial coordinate, and y be the thickness coordinate of the beam as shown in Figure (5-1.1)

Now we will extend the previous theory in chaptes (4) and (5) such that the magnitude of the thermal damping coefficient can be calculated for beams other than Bernoulli - Eulear beams.

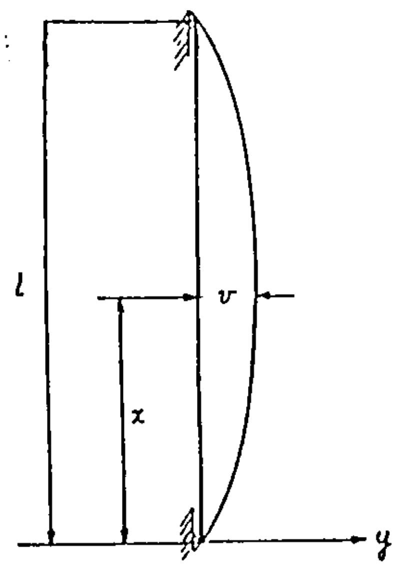


figure 5-1.1

Assume the beam vibrates transversely in x-y plane, U(x,t) and W(x,t) are the centroidal axial and transverse displacement components of the beam, and let $\phi(x,t)$ be the angular rotation of a generic beam cross section. After deformation the displacement components of the beam may be written as [4]

$$u_x = U(x,t) + \phi(x,t).y \text{..........}(5-1.1)$$
$$u_y = v(x,t) = W(x,t) \text{............}(5-1.2)$$

where u_x and u_y are the displacement components in x - y directions, Now let σ_x and τ_{xy} be the axial and shear stresses respectively. Then from the principles of linear thermoelasticity one obtains:

$$\sigma_x = E\,\epsilon_x - E\alpha T = E\left(\frac{\partial u_x}{\partial x} - \alpha T\right) = E\left(\frac{\partial U}{\partial x} + y\frac{\partial \Phi}{\partial x} - \alpha T\right) \text{......}(5-1.3)$$

$$\tau_{xy} = G\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) = G\left(\phi + \frac{\partial w}{\partial x}\right) \text{..................................}(5-1.4)$$

where E is the Young's modulas G is the Shear modulas, α is the coefficient of thermal expansion, and T is the the temperature change from the uniform initial absolute temperature To of the beam. Now assume the following :

N(x,t) axial force along the beam, V(x,t): shear force along the beam, M(x,t) bending moment at section x. The force balance using Figure (5-1.2) yields [4]

$$N(x,t) = \int_{Area} \sigma_x.dA = \int E\left(\frac{\partial U}{\partial x} + y\frac{\partial \phi}{\partial x} - \alpha T\right).dA$$

$$= EA\frac{\partial U}{\partial x} - N_T \text{..................}(5-1.5)$$

$$V(x,t) = \int_A S\,\tau_{xy}.dA = = SAG\left(\phi + \frac{\partial w}{\partial x}\right) \text{.........}(5-1.6)$$

$$M = \int_A s_x . y. dA = \int_A E\left(\frac{\partial ux}{\partial x} - aT\right). y\, dA$$

$$\int_A E\left(\frac{\partial U}{\partial x} + y\frac{\partial \phi}{\partial x} - \alpha T\right). y. dA$$

$$= EI\frac{\partial \phi}{\partial x} - M_T ....5.1.7$$

where S ≅ 0.9 is the shear correction factor, I is the second moment of inertia about y-axis, and A is cross section area of the beam,

$$N_T = E\alpha \int_A T.dA \quad , M_T = E\alpha \int_A yTdA ....5-1.8$$

Noting that N_T, M_T are the thermal axial force and bending moment respectively.

5-2 : EQUATIONS OF MOTION :

From the principles of balance of linear and angular momentums one obtains :

1. summing Forces in x - direction yields:

$$\sum F_x = m\ddot{x}$$

$$\frac{\partial N}{\partial X} - \rho A \ddot{U} = EA\frac{\partial^2 U}{\partial X} - \frac{\partial N_T}{\partial X} - \rho A \ddot{U} = 0...........(5-2.1a)$$

2. Summing forces in y - direction yields :

$$\sum F_y = m\ddot{y}$$

$$\frac{\partial v}{\partial x} - \rho A \ddot{W} = 0$$

or

$$S G A \left(\frac{\partial \phi}{\partial X} + \frac{\partial^2 W}{\partial X^2} \right) - \rho A \ddot{W} = 0 \text{.........} (5-2.1b)$$

By summing moment about any point on the right face of element one obtains :

$$\sum Mo = I\ddot{\Phi}$$

$$\frac{\partial M}{\partial x} - V - \rho I \ddot{\phi} = 0 \text{.........} (5-2.2)$$

$$E I \frac{\partial^2 \phi}{\partial X^2} - \frac{\partial M_T}{\partial X} - S GA \left(\phi + \frac{\partial W}{\partial X} \right) - \rho I \ddot{\phi} = 0 \cdots (5-2.3)$$

Therefore the governing equations of the free vibration of the beam are:

$$E A \frac{\partial^2 U}{\partial X^2} - \frac{\partial N_T}{\partial X} - \rho A \ddot{U} = o \cdots \quad (5-2.4)$$

$$S G A \left(\frac{\partial \phi}{\partial X} + \frac{\partial^2 W}{\partial X^2} \right) - \rho A \ddot{W} = o \qquad (5.2.5)$$

$$E I \frac{\partial^2 \phi}{\partial X^2} - \frac{\partial M_T}{\partial X} - S G A \left(\phi + \frac{\partial W}{\partial X} \right) - \rho I \ddot{\phi} = o \cdots \quad (5-2.6)$$

Where

S is the shear correction factor (S ≈ • 9 for rectangular cross section beams), G is the shear modulas, I is the second moment of inertia of the beam, ρ is the mass density.

[5 - 3] Mechanical boundary conditions :

In this section a general form of mechanical boundary conditions are presented. For mechanical bonndary conditions one must know at $X = o$ and $X = L$ one of the following pairs :

1. Axial Force N or centeriodal axial displacement U.

2. bending moment M or angular rotation of of the center of the beam.

3. shear force V or transverse displacement W.

The general boundary conditions may be written as follows :

$$(i) N(o,t) - K_{uo}U(o,t) = N(L,t) + k_{u1}U(L,t) = 0 \cdots (5-3.1)$$

$$(ii) V(o,t) - k_{wo}W(o,t) = V(L,t) + k_{w1}(L,t) = 0 \cdots (5-3.2)$$

$$(iii) M(o,t) - k_{\phi o}\,\phi(o,t) = M(L,t) + k_{\phi 1}(L,t) = 0 \cdots (5-3.3)$$

where K_{un}, K_{wn}, and $K_{\phi n}(n = o, 1)$ are poistive proportional constants. Each of these constants expresses the relation at the end condition between force and moment with their corresponding displacement and rotation - components. The governing equation between heat condition through the beam when including rotation may be written as follows [7,17]:

$$-K\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial y^2}\right) + r\,C_E\,\dot{T} + Ea\,T\left(\frac{\partial \dot{U}}{\partial X} + y\frac{\partial \dot{f}}{\partial X}\right) = o \cdots (5-3.4)$$

This is known as two way coupled heat conduction equation . For this type of problems Δ T is very small which means that equation (5-3.4) may be written as

$$-K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\rho C_E \dot{T}+E\alpha T_o\left(\frac{\partial \dot{U}}{\partial x}+y\frac{\partial \dot{\phi}}{\partial x}\right)=o\cdots(5-3.5)$$

Note that $\dot{U}=\frac{\partial U}{\partial t}, \dot{\phi}=\frac{\partial \phi}{\partial t}\cdots\cdot$ (5-3.6)

where K is the coefficient heat of thermal conductivity of the beam material, C_E is the specific heat of the beam material at constant deformation, T is the temperature change form uniform initial absolute , temperature. The general form of the thermal boundary conditions may be formulated as follows:

(i) $K\frac{\partial T}{\partial y}\pm K_T T=0,\ y=\pm a\ldots.$ (5-3.7)

(ii) $k\frac{\partial T}{\partial X}\pm K_L T=0,\quad x=0, L\ldots.$ (5-3.8)

where Kn (n = T,L) are the coefficients of surface heat thermal conductivity , and with T = o represents isothermal BC's, K_n= 0 represents thermally insulated BC'S.

5-4 Solution of Longitudinal and Transverse Vibration Eigenvalue problems

In this section and in sections (5-5) and (5-6) a general formulation of the viblation boundary value problems and complete eigen solutions for the flexural vibration eigenvalue problems are presented.

To solve the boundary value problems for rectangular cross section beam , which are formulated by equations in sections (5-1) , (5-2) and

(5-3), it is convenient to introduce the following nondimensional quantities:

$$\xi = \frac{X}{L}\ ,\ \eta = \frac{y}{a}\ ,\ U^* = \frac{U}{L}\ ,\ W^* = \frac{W}{a}$$

$$T^* = \frac{\alpha\ T}{C^2}\ ,c = \frac{a}{L}\ ,t^* = \omega_o . t\ ,\beta = \frac{E\ \alpha^2\ To}{\rho\ C_E}$$

$$t = w_o t_o\ ,\ S = \frac{SGAL^2}{EI}\ V_T = \frac{K_T\ a}{K},\ \ V_L = \frac{K_L a}{k}$$

$$\omega_o = \left(E I\ / \rho\ AL^4 \right)^{1/2}\ ,\ \ \tau_o = \rho\ C_E a^2/K\ \ .... (5\text{-}4.1)$$

where β the thermomechanical coupling parameter.

ωo the reference frequency in radian per second

τo relaxation time of thermal diffusion.and let us assume also

$$U^* = u(\xi)\ e^{\lambda t^*},\qquad W^* = w(\xi)\ e^{\lambda t^*}$$

$$\phi = c\phi(\xi)\ e^{\lambda t^*},\qquad T^* = \theta\ e^{\lambda t^*}\ ....(5-4.2)$$

Now substituting equations (5-4.1) and (5-4.2) into the equations (5-3.1) through (5-3.7) one obtains:

$$u'' - c^2\lambda^2 u - c^2 n_{T'} = o \qquad \text{for} \quad o < \xi < 1.........(5-4.3)$$

$$u'(\xi) - n_T(\xi) \pm k_{u\xi} u_{(\xi)} = o \ \text{ at } \quad \xi = \left(\frac{1}{0}\right)..........(5-4.4)$$

$$s(\phi' + w'') - \lambda^2 w = o$$

$$\phi'' - (s + c^2\lambda^2)\phi - sw' - m'_T = 0 \quad , \text{for } 0 < \xi < 1 \text{ ....(5-4.5)}$$

$$\phi(\xi) + w'(\xi) \pm K_{w\xi} W_{(\xi)} = 0$$

$$\text{at } \xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ ......(5-4.6)}$$

$$\dot{\phi}(\xi) - m_T(\xi) \pm K_{f\xi}\phi(\xi) = 0$$

where

$$n_T(\xi) = \frac{1}{2}\int_{-1}^{1} \theta(\xi,\eta)d\eta \quad , m_T(\xi) = \frac{3}{2}\int_{-1}^{+1} \theta(\xi,\eta)\eta d\eta \text{ .....(5-4.7)}$$

$$\frac{\partial\theta}{\partial\eta} \pm V_T\theta = 0 \quad , \text{at } \eta = \pm 1 \quad \text{.......... (5-4.8)}$$

$$\frac{\partial\theta}{\partial\xi} \pm v_L\theta = 0 \quad , \text{at } \xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{.......... (5-4.9)}$$

where the prime indicates differentiations of a function with respect to its argument. Starting with coupled heat conduction one obtains:

$$-K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \rho C_E \dot{T} + E\alpha T_o\left(\frac{\partial \dot{U}}{\partial x} + y\frac{\partial \dot{\phi}}{\partial x}\right) = 0.0 \text{ ......(5-4.10a)}$$

and

$$\xi = \frac{x}{L} \rightarrow \frac{\partial\xi}{\partial x} = \frac{1}{L} \quad , \eta = \frac{y}{a} \rightarrow \frac{\partial\eta}{\partial y} = \frac{1}{a} \quad \text{....... (5-4.10b)}$$

$$T = \frac{c^2}{\alpha}\theta \cdot e^{\lambda t^*} = \frac{c^2}{\alpha}\theta - e^{\lambda\omega ot}$$

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial \xi}\cdot\frac{\partial \xi}{\partial x} = \frac{\partial T}{\partial \xi}\cdot\frac{1}{L} \quad \text{....... (5-4.10c)}$$

5-5 Case of Rectangular Cross section Beams:

The governing equations (5-4.1) up to (5-4.11) are now to be solved A solution of coupled heat conduction equation (5-4.11) is sought as follows:

$$\theta = X(\xi,\eta) + \eta Y(\xi) + Z(\xi) \text{.........}(5-5.1)$$

$$\frac{\partial \theta}{\partial \xi} = \frac{\partial^2 x}{\partial \xi} + \eta \frac{\partial Y}{\partial \xi} + \frac{\partial z}{\partial \xi}$$

$$\frac{\partial^2 \theta}{\partial \xi^2} = \frac{\partial^2 x}{\partial \xi^2} + \eta \frac{\partial^2 Y}{\partial \xi^2} + \frac{\partial^2 z}{\partial \xi^2} \text{............}(5-5.2)$$

$$\frac{\partial \theta}{\partial \eta} = \frac{\partial x}{\partial \eta} + Y(\xi)$$

$$\frac{\partial^2 \theta}{\partial \eta^2} = \frac{\partial^2 x}{\partial \eta^2} \text{..............................}(5-5.3)$$

Now substituting equation (5-5.1), (5-5.2) into (5-4.11) yields:

$$C^2\left(\frac{\partial x^2}{\partial \xi^2} + h\frac{\partial^2 Y}{\partial \xi^2} + \frac{\partial^2 z}{\partial \xi^2}\right) + \frac{\partial^2 x}{\partial \eta^2} - \lambda\tau\left(X + \eta Y + Z\right) = \beta\lambda\tau(U'/C^2 + \eta\phi)$$

..... (5-5.4)

Next breaking (5-5.4) into the following equations:

$$C^2 \frac{\partial x^2}{\partial \xi^2} + \frac{\partial x^2}{\partial \eta^2} - \lambda\tau X = 0 \quad \text{for } 0 < \xi < 1, -1 < \eta < 1 \text{......}(5-5.5)$$

$$C^2 \frac{\partial^2 Y(\xi)}{Y\xi^2} - \lambda\tau \; Y(\xi) = \beta\lambda\tau \; \phi'(\xi) \quad \text{for } 0 < \xi < 1 \text{......}(5\text{-}5.6)$$

$$C^2 \frac{\partial^2 Z(\xi)}{\partial \xi^2} - \lambda\tau Z(\xi) = \beta\lambda\tau \frac{U'(\xi)}{C^2} \text{......}(5-5.7)$$

For the mechanicand and the thermal boundary conditions , and from equations (5-4.8) and (5-4.9), one has:

$$\frac{\partial\theta}{\partial\eta} \pm V_T\theta = 0, \quad \text{at} \quad \eta = \pm 1 \quad ....(5\text{-}5.8)$$

$$\frac{\partial\theta}{} \pm V_T\theta = 0 \quad \text{at} \quad \} = \left(\frac{1}{0}\right) ....(5-5.9)$$

$$\theta = X(\},\eta) + \eta Y(\}) + Z(\}) ....(5-5.10)$$

$$\frac{\partial\theta}{\partial\eta} = \frac{dx}{d\eta} + Y(\}) \quad — \quad .... (5\text{-}5.11)$$

$$\frac{\partial\theta}{\partial\}} = \frac{dx}{d\}} + \eta\frac{\partial Y}{\partial\}} + \frac{dz}{\partial\}} ....(5-5.12)$$

Substituting equations (5-5.10), (5-5.11) and (5-5.12) into (5-5.8) one obtains:

$$\frac{dx}{d\eta} + Y(\}) \pm V_T(x + \eta Y + Z) = o \quad ....(5\text{-}5.13)$$

$$\frac{dx}{d\}} + \eta\frac{dY}{d\}} + \frac{dz}{d\}} \pm V_L(X + \eta Y + Z) = o \quad ....(5\text{-}5.14)$$

From (5-5.14) one gets

$$\frac{dx}{d\}} \pm V_L X = 0 \quad \text{at } \} = \left(\frac{1}{0}\right) \quad ....(5\text{-}5.15)$$

$$\frac{dY}{d\}} \pm V_L Y = 0 \quad \text{at } \} = \left(\frac{1}{0}\right) ....(5-5.16)$$

$$\frac{dZ}{d\}} \pm V_L Z = 0 \quad \text{at } \} = \left(\frac{1}{0}\right) ....(5-5.17)$$

And from (5-5.13) one obtains

$$\frac{dx}{d\eta} \pm V_T X = -(1+V_T)\,Y(\xi) \pm V_T Z(\xi) \quad \text{at} \qquad \xi = \ne 1 \ldots\ldots(5-5.18)$$

Also write equation (5-4.7) as follows:

$$n_T(\xi) = \frac{1}{2}\int_{-1}^{+1} \theta(\xi,\eta)\,d\eta \qquad , \; m_T(\xi) = \frac{3}{2}\int_{-1}^{+1} \theta(\xi,\eta)\,\eta\,d\eta$$

$$n_T(\xi) = \frac{1}{2}\int_{-1}^{+1} [X(\xi,\eta) + \eta Y(\xi) + Z(\xi)]\,d\eta$$

$$n_T = \frac{1}{2}\int_{-1}^{+1} x(\xi,\eta)\,d\eta + z(\xi) \qquad \ldots\ldots.(5-5.19)$$

$$m_T(\xi) = \frac{3}{2}\int_{-1}^{+1} [X + \eta Y + Z].\eta\,d\eta$$

$$m_T(\xi) = \frac{3}{2}\int_{-1}^{+1} X(\xi,\eta)\,\eta\,d\eta + Y(\xi) \ldots\ldots.(5-5.20)$$

5-6 Solution of thermal part of eigenvalue problem:

Now equations (5-5.5) ,(5-5.6), (5-5.7), are solved to the boundary conditions in equations (5-5.15) and (5-5.16) and (5-5.17) i.e :

$$C^2 \frac{\partial^2 x}{\partial \xi^2} + \frac{\partial^2 x}{\partial \eta^2} - \lambda \tau x = 0 \quad \text{for} \quad 0 < \xi < 1, -1 < \eta < 1 \ldots\ldots(5-6.1)$$

$$\frac{\partial x(\xi,\eta)}{\partial \xi} \pm V_L(\xi,\eta) = 0 \qquad \text{at } \xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ldots\ldots\ldots.(5-6.2)$$

Using the standard separation of variable techinques one assumes

$$X(\xi,\eta) = X_1(\xi) \bullet X_2(\eta) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5\text{-}6.3)$$

Substituting equation (5 - 6.3) into equation (5 - 6.1) yields:

$$C^2 X''_1 X_2 + X_1 X''_2 - \lambda\tau X_1 X_2 = 0 \ldots\ldots.(5-6.4)$$

Now rewrite (5 - 6.4) as follows

$$\frac{X''_1}{X_2}+\frac{X''_2}{C^2X_2}-\frac{\lambda\tau}{C^2}=0$$

or

$$\frac{X''_1}{X_1}=\alpha_n^{\ 2}=\frac{\lambda\tau}{C^2}-\frac{X''_2}{C^2X_2}\ \cdots\ (5-6.5)$$

or :

$$X_1(\xi)=A_{1n}\cos\alpha_n\xi+B_{1n}\sin\alpha_n\xi\ ......(5-6.6)$$

where $A_{1n,}B_{1n}$ are constants to be determined from the boundary conditions. Also from equation (5 - 6.5) one has:

$$-\lambda\tau+\frac{X''_2}{X_2}=\ +C^2\alpha_n^2$$

or:

$$\frac{X''_2}{X_2}=\left(\lambda\tau\ +C^2a_n^2\right)=\mu^2{}_n$$

$$X_2(\eta)=\ An\cosh\mu^2{}_n\ +\ Bn\sinh\ \mu_n........(5-6.7)$$

where An, Bn are constants to be determined from equation (5-5.18) by using Fourier integrals. Note that

$$\mu n=\left(\lambda\tau+C^2\alpha n^2\right)^{\frac{1}{2}}\ ....\ (5\text{-}6.8)$$

Now go back to equation 5-6.6 and substituting into (5-6.2) one finds that at $\xi=0$

$$A_{1n}=\alpha_n,\ B_{1n}=\nu_L\ \ ....\ (5\text{-}6.9)$$

The general solution of equation (5-6.1) may be written in the form:

$$X(\xi,\eta)=\sum_{n=0}^{\infty}(An\ \mathrm{Cosh}\ \mu_n.\eta+\ B_n\mathrm{Sinh}\ \mu_n.\eta\}(\alpha_n\cos\alpha_n\xi+n_L\sin\alpha_n\xi)$$

$$.................\ (5\text{-}6.10)$$

To find α_n values go back to equation (5-6.2), at $\xi=1$, one obtains

$$X_l(\}) = (\alpha_n \text{Cos } \alpha_n \} + V_L \text{ Sin } \alpha_n \}) ....(5-6.11)$$

$$\frac{\partial x_l(\})}{\partial \}} \pm V_L X_l(\}) = 0 \quad \text{at } \} = 1 \quad ....(5-6.12)$$

Substituting (5-6.11) into (5-6.12) leads to at $\} = 1$

$$-\alpha_n^2 \text{ Sin } \alpha_n + \alpha_n V_L \cos \alpha_n + V_L[\alpha_n \cos \alpha_n + V_L \sin \alpha_n] = 0$$

$$-\alpha^2 n \text{ Sin } \alpha_n + 2\,\alpha_n V_L \cos \alpha_n + V_L^2 \text{Sin } \alpha_n = 0 \quad ....(5-6.13)$$

Dividing equation (5-6.13) by [cos α_n] leads:

$$-\alpha_n^2 \tan \alpha n + 2\,\alpha_n V_L + V_L^2 \tan \alpha_n = 0.0$$

or

$$\tan \alpha_n = \frac{2V_L.\alpha_n}{\alpha_n^2 - V_L^2} \quad ....5-6.14$$

$\alpha_0, \alpha_1, \alpha_2, ...\alpha_n$ are the roots of equation (5-6.14). If $V_L = o$ this means that $\tan \alpha_n = o$ or $\sin \alpha_n = o$ i.e

$$\alpha_n = n\pi \quad .... 5-6.15$$

$$\alpha_n = \pi, 2\pi, 3\pi, ...$$

Now substituting of equation (5-6.10) into (5-5.19) yields

$$n_T(\}) = \frac{1}{2}\int_{-1}^{+1} x(\}, \eta) d_h + z(\})$$

$$= \frac{T}{2}\int_{ln}^{l} \sum_{o}^{\infty} (An \text{ Cosh } \mu_n.\eta + Bn \sinh \mu_n.\eta)(\alpha_n.\cos \alpha_n\} + V_L \sin\alpha_n\}) + Z(\})$$

$$n_T(\}) = Z(\}) + \sum_{n=o}^{\infty}\left(\frac{An \text{ Sin } h\mu_n\eta}{\mu n}\right)(\alpha_n \cos \alpha_n\} + V_L \text{ Sin } \alpha_n\}) ....(5-6.16)$$

Also substituting equation (5-6.1) into (5-5.20) leads to

$$m_T(\}) = \frac{3}{2}\int_{-1}^{+1} X(\},\eta)\eta d\eta + Y(\})$$

$$= Y(\}) + \frac{3}{2}\int_{-1}^{+1}\sum_{n=o}^{\infty}[An\,Cosh\;\mu_n.\eta + Bn\,sinh\;\mu_n.\eta][\alpha_n \cos\alpha_n\} + V_L \sin\alpha_n\}]\eta.d\eta$$

$$m_T(\}) + \sum_{n=o} 3Bn\left(\frac{Cosh\;\mu_n}{\mu_n} - \frac{sinh\;\mu_n}{\mu_n^{\;2}}\right)\times(\alpha_n\,Cos\,\alpha_n\} + V_L\,sin\alpha_n\})......(5-6.17)$$

$$\frac{\partial\,x(\},\eta)}{\partial\,\eta} \pm V_T X(\},\eta) = -(1+V_T)\,Y(\}) \pm V_T Z(\}),\ \text{at}\ \eta = \pm 1$$

$$\sum_{n=o}^{\infty}[An\,\mu n\,sinh\;\mu n\eta + Bn\mu_n\,cosh\;\mu n\eta][\alpha_n\cos\alpha_n\} + V_T\sin\alpha_n\}]\pm$$

$$V_T.\sum_{n=o}^{\infty}[An\,cosh\;\mu n\eta + Bn\,sinh\;\mu_n\eta][\alpha_n\cos\alpha_n\} + V_L\sin\alpha_n\}]$$

$$= -(1+V_T)\,Y(\}) \pm V_T Z(\})\qquad at\eta = \pm 1\qquad ....(5-6.18)$$

Addition and subtraction of two resulting equation of equation (5- 6.18) will result in

$$\sum_{n=o}^{\infty} An(\mu\,n\,sinh\,\mu\,n + v_T\,cosh\,\mu n)\times(\alpha_n cos\,\alpha_n\} + V_L\,sin\,\alpha_n\}) = -v_T\,Z(\})\ ...(5-\ 6.19)$$

$$\sum_{n=o}^{\infty} Bn(\mu\,n\,cosh\,\mu\,n + v_T\,sinh\,\mu n)(\alpha_n\cos\alpha_n\} + V_L\,sin\,\alpha_n\}) = -(1+v_T).Y(\})...(5-6.20)$$

5-7 EIGEN SOLUTION FOR FREE TRANSVERSE VIBEATION BOUNDARY-VALUE PROBLEM

Now consider the previous beam with lateral surfaces thermally insulated and end surfaces kept at constant temerature. To begin with frist substitute equation (5-6.17) into equations (5- 4.5) and (5-4.6) to eliminate m_T leads to the following system of differential equations.

$$m_T(\}) = Y(\}) + \sum_{n=o}^{\infty} 3\, Bn\left(\frac{\cosh\mu n}{\mu n} - \frac{\sinh \mu n}{\mu n}\right) \times (\alpha_n \cos \alpha_n x + V_L \cdot \sin \alpha_n \})...(5-7.1)$$

From (5-4.5) and (5-4.8) we have :

$$\phi'' - \left(s + c^2 \lambda^2\right)\phi - sw' - m'_T = o \quad \text{for} \quad o < x > 1 .$$

$$\phi(\}) + w'(\}) \pm k_{w\}}\, w(\}) = o \ \text{at}\ \} = \left(\frac{1}{o}\right)$$

$$\phi'(\}) - m_T(\}) \pm k_{\phi\}}\, \phi(\}) = o$$

The general Solution of the above second order linear differential equations with zero shear force at bothends and totation is given by

$$\phi(\}) = \sum_{n=o}^{\infty} B_n^{*}(\alpha_n \sin \alpha_n \} - V_L \cos \alpha_n \})...(5-7.2)$$

$$W(\}) = +\sum_{n=o}^{\infty} \frac{B_N^{*} S \alpha_n (a_n \operatorname{Cos} \alpha_n \} + V_L \operatorname{Sin} \alpha_n \})}{\left(S\ \alpha_n^2 + \lambda^2\right) g_n} .....(5-7.3)$$

$$Y(\}) = -\sum_{n=o}^{\infty} B_n^{*} \cdot \left(\frac{\beta\lambda\tau\alpha_n}{\mu_n^2}\right)(\alpha_2 \operatorname{Cos} \alpha_n \} + V_L \sin \alpha_n \})...(5-7.4)$$

$$f_n(\lambda) = 3(\mu_n \operatorname{Cosh} \mu_n - \sinh \mu_n) / \mu_n^2 ....(5-7.5)$$

$$g_n(\lambda) = S\lambda^2 / \left(S\ \alpha_n^2 + \lambda^2\right) + \alpha_n^2\left(1 + \beta\lambda\tau / \mu\ n^2\right) + c^2\lambda^2 ......(5-7.6)$$

Substitution of equation (5- 7.4) into equation (5-6.20) gives

$$\sum_{h=0}^{\infty} B_n h_n(\alpha_n \cos \alpha_n \xi + V_L \sin \alpha_n \xi) = -(1+V_T)\sum_{K=1}^{3}$$

$$\left[\frac{\beta\lambda\tau \Omega_K}{C^2\Omega_K - \lambda T}\right] \times (C_K \sinh \Omega_k \xi + C_{K+3} \operatorname{Cosh} \Omega_k \xi) \ldots (5-7.7)$$

where ($h_n = o$) is the characteristic equation of the system. The roots of the characteristic equation are the eigenvalues of the system. The general form of the characteristic equation of the system of equations is :

$$\alpha_n^2 [1 + F_A] + \text{Function}(\lambda, n\pi c) = 0.0 \ldots\ldots\ldots\ldots (5\text{-}7.8)$$

where f_Λ is the thermomechanical coupling function will be determined later in chapter 6 and 7,and the charecteristic equation has the form:

$$h_n = \mu_n \cosh \mu_n + V_T \operatorname{Sinh} \mu_n - P_n \ldots\ldots\ldots (5-7.9)$$

where

$$P_n = (1+V_T)\beta \lambda \tau \alpha_n^2 F_n / \mu n^2 g_n \ldots (5-7.10)$$

CHAPTER SIX

THERMO ELASTO PLASTIC DAMPING COEFFICIENT CALCULATIONS WITH FREE ENDS FOR SIMPLY SUPORTED BEAM.

6 -1 INTRODUCTION

In this chapter the coupled heat conduction equation is solved and the magnitude of the thermomechanical coupling function that arises from the characteristic equation of the system is presented. The magnitude of thermo elastoplastic damping coefficient is obtained in a closed form coresponding to the adiabatic , mixed and isothermal boundary conditions.

6-2 ADIABATIC THERMAL DAMPING COEFFICIENT

[For nc=o]

Now return back to equations (5- 7.9) and (5-7.10) one has the characteristic equation h_n :

$$h_n=\mu_n \cosh\mu_n+V_T \sinh\mu_n-P_n ....(6-2.1)$$

$$P_n=(1+V_T)\beta\, \lambda\, \tau\, \alpha_n^2\, f_n/\, \mu_n^2\, g_n ......(6-2.2)$$

$$\mu_n=\left(\lambda\tau+C^2\alpha_n^2\right)^{1/2} ....(6-2.3)$$

let V_T= o.o, and from equation (5-6.14) one has for the adiabatic boundary conditions

$$\alpha_n = n\pi \text{ .... (6-2.4)}$$

This means that equation (6-2.1) can be written as :

$$h_n=\mu_n \cosh \mu_n-\frac{\beta\lambda T(h\pi)^2 . F_n}{\mu n^2 . g_n} \quad (6-2.5)$$

From equation (5-7.6) and equation 5-7.7 one obtains :

$$f_n= 3 (\mu_n \cosh \mu_n-\sinh \mu_n)/\mu n^2 \quad (6-2.6)$$

$$g_n(\lambda) = s\lambda^2/(s\alpha_n^2+\lambda^2)+\alpha_n^2\left(1+\beta\lambda T/\mu n^2\right)+C^2\lambda^2$$

$$g_n(\lambda)=\left[s\lambda^2/(Sn^2\pi^2+\lambda^2)\right]+n^2\pi^2\left(1+\beta\lambda\tau/n^2\right)+C^2\lambda^2 \quad (6-2.7)$$

Substituting into equation (6-2.1) leads to:

$$hn=\mu n \cosh \mu n-\frac{\beta\lambda\tau(n\pi)^2 * 3(\mu n \cosh \mu n-\sinh \mu n)}{\mu n^2 * \mu n^2\left\{\left[\frac{s\lambda^2}{s(n\pi)^2+\lambda^2}\right]+(n\pi)^2\left(\frac{1+\beta\lambda\tau}{\mu n^2}\right)+C^2\lambda^2\right\}} \quad (6-2.8)$$

Now set $h_n = o.o$ ....(6-2.9)

From equation (6-2.9) and after manipulations one gets :

$$\left\{\left[\frac{s\lambda^2}{s(n\pi)^2+\lambda^2}\right]+(n\pi)^2\left(1+\frac{\beta\lambda\tau}{\mu n^2}\right)+C^2\lambda^2\right\} * \mu n^5 \cosh \mu n = 3.\beta\lambda\tau(n\pi)^2(\mu n \cosh \mu n-\sinh \mu n)$$

Now divided by $\cosh \mu_n$ yields

$$\left\{\left[\frac{s\lambda^2}{s(n\pi)^2+\lambda^2}\right]+(n\pi)^2\left(1+\frac{\beta\lambda\tau}{\mu n^2}\right)+C^2\lambda^2\right\}\mu n^5 = 3\beta\lambda\tau(n\pi)^2(\mu n-\tanh \mu n) \quad (6-2.10)$$

After manipulation of equation (6-2.10), one can write it as follows :-

$$(n\pi)^2\left[1+F_B(\lambda\tau)\right]+\left[\frac{S\lambda^2}{S(n\pi)^2+\lambda^2}\right]+C^2\lambda^2= 0.0 \quad (6-2.11)$$

where $F_B(\lambda\tau)$ is the thermomechanical coupling function given by

$$F_B(\lambda\tau, nc)=\left[\frac{\beta\lambda\tau}{\mu n^2}\right]*\left\{1-\left(\frac{3}{\mu n^2}\right)*\left(1-\frac{\tanh \mu n}{\mu n}\right)\right\} \quad (6-2.12)$$

where $\mu n = \left[(cn\pi)^2+\lambda\tau\right]^{\frac{1}{2}}$

Now when the thickness of the beam c becoming very small i,e $c \rightarrow o.o$, the thermomechanical coupling function F_B can be written as

$$F_B(\lambda\,\tau,o) = Fo(\lambda\,\tau) = \frac{\beta\, T\, \lambda}{\mu^2\, n}\left[1 - \frac{3}{\mu^2\, n}\left(1 - \frac{\tanh\, \mu\, n}{\mu\, n}\right)\right]$$

Next the magnitude of the thermal damping coefficient calculated when $nc \rightarrow 0.0$ that is $\mu_n = \sqrt{\lambda\tau}$

Substituting the value of μn into $\mu_o(\lambda\tau)$ gives

$$F_B(\lambda\,\tau,o) = F_o(\lambda\,\tau) = \beta\left[1 - \frac{3}{\lambda\, T}\left(1 - \frac{\tanh\sqrt{\lambda\,\tau}}{\sqrt{\lambda\,\tau}}\right)\right] \text{......} \ (6\text{-}2.13)$$

where

β : the thermomechanical coupling parameter

ωo : the reference frequancy

τo : the relaxation time of thermal diffusion

$$\beta = \frac{E\,\alpha^2\, To}{\rho C_E},\quad \omega_o = \left(\frac{EI}{\rho\, AL^4}\right)^{1/2},\quad \tau_o = \frac{\rho C_E\, a^2}{K} \text{......} (6-2.14)$$

Now under harmonic motion $\lambda = i\omega$ $(\omega^2 > o)$, F_B is complex function , The real part of this function represents stiffening effect , and the imaginary part represents a damping effect on the vibrated system. Assume g_o as wave length parameter , defined as

$$g_o = \frac{1}{nc} = \frac{1}{n.a/L} = \frac{L}{na} \quad \text{...............} \ (6\text{-}2.15)$$

Then

$$\tanh(1+i)\Lambda = \frac{\sinh 2\Lambda + i\sin 2\Lambda}{2[1+\sinh^2\Lambda - \sin^2\Lambda]} \text{.......} (6-2.16)$$

From equation (6-2.6) we have the following is obtained

$$Fo(\lambda\tau) = \beta\left[1 + \frac{3i}{\omega\tau} - \frac{3}{\sqrt{2(\omega\tau)^{3/2}}} * (1+i) * \frac{\sinh_2 + i\sinh_2 A}{2[1+\sinh^2 A - \sin^2 A]}\right] \text{.....} (6-2.17)$$

The imaginary part of equation (6-1.17) is the thermo-elasto-plastic damping coefficient i.e $\frac{G_\beta}{\beta}$, and the real part of equation (6-1.17) represents stiffeness effect i.e

$$Re[Fo(\lambda\tau)] = \beta\left[1 - \frac{3}{\sqrt{2}(\omega\tau)^{3/2}} * \frac{(\sinh_2 A - \sin_2 A)}{2\left[1 + \sinh^2 A - \sin^2 A\right]}\right] \ldots\ldots(6-2.18)$$

Thus the magnitude of the thermal damping coefficient for Bernoulli - Eulear beam i .e nc = o is

$$\frac{G_{Bo}}{\beta} = \frac{3}{\omega\tau} - \frac{1.061}{|\omega\tau|^{3/2}} * \frac{\sinh\sqrt{2\omega\tau} + \sin\sqrt{2\omega\tau}}{1 + \sinh^2\sqrt{\frac{\omega\tau}{2}} - \sin^2\sqrt{\frac{\omega\tau}{2}}} \ldots\ldots\ldots\ldots(6-2.19)$$

6-3 ADIABATIC THERMAL DAMPING COEFFICIENT FOR NC NOT EQUAL ZERO

In the previous section the magnitude of thermal damping coefficient for Bernoulli-Elear beam is obtained, Now in this section the magnitude of the thermal damping coefficient for beams with nc ≠ o will be obtained. Form equation (6-2.12) one has

$$F_B(\lambda\tau, nc) = \frac{\beta\lambda\tau}{\mu^2 n}\left[1 - \frac{3}{\mu^2 n} + \frac{3\ \tanh\mu n}{\mu^3 n}\right] \ldots\ldots\ldots\ldots\ldots (6\text{-}3.1)$$

$$\text{where } \mu n = \left[(n\pi c)^2 + \lambda\tau\right]^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots (6\text{-}3.2)$$

Substituting

$$\lambda = i\omega \quad (\omega^2 > o) \ldots\ldots\ldots\ldots\ldots\ldots (6\text{-}3.3)$$

leads to:

$$\mu n = \left[(n\pi\ c)^2 + i\omega\tau\right]^{1/2}$$

$$\mu n = \left[(n\pi c)^4 + (\omega\tau)^2\right]^{\frac{1}{4}} * \left[\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta\right] \ldots(6-3.4)$$

$$= A\left[\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta\right]$$

where:

$$\theta = \tan^{-1}\left[\frac{\omega\tau}{(n\pi c)^2}\right], \quad A = \left[(n\pi c)^4 + (\omega\tau)^2\right]^{\frac{1}{4}} \ldots\ldots\ldots\ldots\ldots (6.3.5)$$

Now substituting equation (6-3.4) into equation (6-3.1) yields:

$$F_B(\lambda\tau, nc) = \frac{\beta\lambda\tau}{\mu^2 n} - \frac{3\beta\lambda\tau}{\mu^4 n} + \frac{3\beta\lambda\tau \tanh \mu n}{\mu^5 n}$$

$$= \frac{\beta\omega\tau i}{A^2\left[\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta\right]^2} - \frac{3\beta\omega\tau i}{A^4\left[\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta\right]^4}$$

$$+ \frac{\beta i\omega\tau \tanh|A|\left[\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta\right]}{A^5\left[\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta\right]^5} \quad \ldots\ldots\ldots(6-3.6)$$

$$\left[\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta\right]^2 = \cos\theta + i\sin\theta \quad \ldots\ldots(6-3.7a)$$

$$\left[\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta\right]^4 = \cos 2\theta + i\sin_2\theta \quad \ldots\ldots(6-3.7b)$$

$$\frac{i}{\cos\theta + i\sin\theta} = \sin\theta + i\cos\theta \quad \ldots\ldots\ldots 6-3.7c$$

$$\tanh|A|\left[\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta\right] = \frac{\cos Aq \sinh A\rho + i\sin Aq \cosh Ap}{\cos Aq \cosh Ap + i\sin Aq \sinh AP} \ldots\ldots(6-3.8)$$

where $p = \cos\frac{1}{2}\theta$, $q = \sin\frac{1}{2}\theta$

$$\tanh A[p + iq] = \frac{\sinh^2 Aq + i\sin^2 AP}{2\left[1 + \sinh^2 AP - \sin^2 Aq\right]} \ldots\ldots\ldots\ldots(6-3.9)$$

Substituting equations (6-3.7) through (6-3.9) into equation (6-3.6) one obtains:

$$F_B(\lambda\tau, nc) = \frac{[\omega\tau]^2 + i\omega\tau[cn\pi]^2}{[cn\pi]^2 + [\omega\tau]^2} - 3\frac{2[n\pi c]^2 + i[n\pi c]^4\omega\tau - (\omega\tau)^3}{\left[[n\pi c]^4 + [\omega\tau]^2\right]^2}$$

$$+ \frac{3i\beta\tau}{\left[[n\pi c]^4 + [\omega\tau]^2\right]^{5/2}} \bullet \frac{[R_1 + iR_2]}{R_3} \text{..............................} (6-3.10)$$

where:

$$R_1 = \text{Sinh}\, 2Aq\left[\sin 2\theta \cos\frac{1}{2}\theta + \cos 2\theta \sin\frac{1}{2}\theta\right] +$$

$$+ \sin 2AP\left[\cos 2\theta\ \cos\frac{1}{2}\theta - \sin 2\theta \sin\frac{1}{2}\theta\right]$$

$$R_2 = \sin 2AP \quad R_1 = \text{Sinh}\, 2Aq\left[\sin 2\theta \cos\frac{1}{2}\theta + \cos 2\theta \sin\frac{1}{2}\theta\right] +$$

$$-\sinh 2Aq \quad \sin 2AP\left[\cos 2\theta\ \cos\frac{1}{2}\theta - \sin 2\theta \sin\frac{1}{2}\theta\right]$$

$$R_3 = 2[1 + \sinh^2 AP - \sin^2 Aq]\left[R_1 = \text{Sinh}\, 2Aq\left[\sin 2\theta \cos\frac{1}{2}\theta + \cos 2\theta \sin\frac{1}{2}\theta\right]^2\right]$$

$$+ \sin 2AP\left[\cos 2\theta\ \cos\frac{1}{2}\theta - \sin 2\theta \sin\frac{1}{2}\theta\right] \qquad \text{................} (6-3.11)$$

Now the magnitude of the thermo-mechanical coupling function $F_B(\lambda\tau, nc)$ which is obtained from equation (6-3.10) can be sperated into real part and imaginary part. The imaginary part is $\frac{G_\beta}{\beta}(\omega\tau, nc)$ which represents the magnitude of the thermal damping coefficient i.e.

$$\frac{G_\beta}{\beta} = \frac{\omega\tau[cn\pi]^2}{[cn\pi]^4 + [\omega\tau]^2} - 3\frac{[n\pi c]^4 . \omega\tau - [\omega\tau]^3}{\left[[n\pi c]^4 + [\omega\tau]^2\right]^2} + \frac{3\omega\tau R_1}{R_3\left[[n\pi c]^4 + [\omega\tau]^2\right]^{5/2}} \text{....} (6\text{-}3.12)$$

Note that for nc = 0 equation (6-3.12) reduce to equation (6-2.19).

6-4 MIXED THERMAL DAMPING COEFFICIENT:

In this section the magnitude of the thermal damping coefficient is calculated corresponding to the mixed thermally boundary conditions i.e lateral surfaces of the beam are thermally insulated and the end surfaces are kept at constant temperature. In this case the thermall boundary conditions are:

$$K_T \bullet T = 0 \quad \text{at } y = \pm a$$
$$k\frac{\partial T}{\partial x} = \quad \text{at } x = 0, L \qquad \text{.................. (6-4.1)}$$

or

$$V_T.\theta = 0, \quad \text{at } \eta = \pm 1$$
$$\frac{\partial \theta}{\partial \}} = 0 \quad , \text{ at } \} = 1,0 \qquad \text{..................... (6-4.2)}$$

The solution of heat conduction equation subjected to the above boundary condition will give the following characteristic equation.

$$hn = 0.0 = \mu n \cosh \mu n + V_T \sinh \mu n - \frac{(1+V_T)\beta\lambda\tau(n\pi)^2[\mu n \cosh \mu n - \sinh \mu n]}{\mu n^4\left[\frac{S\lambda^2}{[S(n\pi)^2+\lambda^2]} + (n\pi)^2\left(1+\frac{\beta\lambda T}{\mu^2 n}+\frac{c^2\lambda^2}{3}\right)\right]}$$

.............(6-4.3)

Solving this characteristic equation for the thermomechanical coupling function F_B leads to:

$$\frac{F_B}{B} = \frac{\lambda\tau}{\mu_n^3} - 3\,\frac{3(1+V_T)\lambda\tau.(\mu n \cosh \mu n - \sinh \mu n)}{\mu n^4[\mu n \cosh \mu n + V_T \sinh \mu n]} \qquad \text{...........(6-4.4)}$$

$$\mu n = \left[(n\pi c)^2 + i\omega\tau\right]^{\frac{1}{2}}, \qquad \lambda = i\omega \qquad \text{................ (6-4.5)}$$

substituting equation (6-4.5) into equation (6-4.2) one obtains

$$\frac{F_\beta}{\beta}=\frac{(\omega\tau)^2+i\omega\tau(n\pi c)^2}{(n\pi c)^4+(\omega\tau)^2}+\frac{Z_5[Z_1Z_4-Z_2Z_3]-iZ_5[Z_1Z_3+Z_2Z_4]}{Z_1^2+Z_2^2}$$

..........(6-4.6)

The imaginary part of equation (6-4.6) is the thermoelastoplastic damping damping coefficient $\frac{G_\beta}{\beta}$ given by:

$$\frac{G_\beta}{\beta}=\frac{\omega\tau(n\pi c)^2}{(n\pi c)^4+(\omega\tau)^2}-\frac{Z_5[Z_1Z_3+Z_2Z_4]}{Z_1^2+Z_2^2} \qquad \text{...... (6-4.7)}$$

where

$A=\left[(\omega\tau)^2+(n\pi c)^4\right]^{\frac{1}{4}}$, $P=\cos\frac{1}{2}\theta$ $\quad ,q=\sin\frac{1}{2}\theta$

$,\theta=\tan^{-1}\left[\frac{\omega\tau}{(n\pi c)^2}\right]$

$M=\cos Aq.\cosh Ap \quad ,N=\sin Aq.\ \sinh AP$

$A_1=\left[(n\pi c)^4+(\omega\tau)^2\right]^{\frac{5}{4}}$, $P_1=COS\frac{5}{2}\theta$, $q_1=\sin\frac{5}{2}\theta$

$R_1=V_T.\left[(n\pi c)^4-(\omega\tau)^2\right]$, $R_2=2\omega\tau\, v_T\,[n\pi c]^2$

$D_1=\sinh AP\bullet\cos Ag\cdot$, $D_2=\sin Ag\bullet\cosh AP Z_1=A_1[P_1M-q_1N]+[R_1D_1-R_2D_2]$

$Z_2=A_1[P_1N+q_1M]+[R_1D_2+D_1R_2]$

$Z_3=A[PM-qN]-D_1$

$Z_4=A[PN+qM]-D_2$ ....(6-4.8)

$Z_5=3(1+V_T)\omega\tau$

6-5 ISOTHERMAL THERMAL DAMPING COEFFICIENT:

In this section the magnitude of the thermal damping coefficient is obtained corresponding to isothermal boundary condition. In this case the boundary conditions are:

$$\begin{matrix} T = o & \text{at } y = \pm a \\ T = o & \text{at } x = o, L \end{matrix} \quad \text{.........} (6\text{-}5.1)$$

or

$$\begin{matrix} \theta = o & \text{at } \eta = \pm 1 \\ \theta = o & \text{at } \xi = o, 1 \end{matrix} \quad \text{..........} (6\text{-}5.2)$$

The solution of heat conduction equation under above boundary conditions will give the characteristic equation as:

$$hn = 0.0 = \mu n \cosh \mu n + \nu_T \sinh \mu n - \frac{(1+\nu_T)\beta\lambda\tau\alpha^2{}_n[\mu n \cosh \mu n - \cosh \mu n]}{\mu n^4\left[\frac{s\lambda^2}{s(\alpha_n)^2+\lambda^2} + \alpha n^2\left(1+\frac{\beta\lambda\tau}{\mu n^2}+\frac{c^2\lambda^2}{3}\right)\right]}$$

where $\mu n = \left[c^2\alpha^2{}_n + \lambda\tau\right]^{\frac{1}{2}}$ .... (6-5.4)

$$\tan \alpha_n = 2\nu_L.\frac{\alpha_n}{\alpha n^2 - \nu_L{}^2} \quad \text{.............} (6\text{-}5.5)$$

Solving equation (6-5.3) for the thermomechanical coupling function $F_{\beta\lambda}$ will give:

$$\frac{F\beta}{\beta} = \frac{\lambda\tau}{\mu n^2} - \frac{3(1+\nu_T)\lambda\tau.(\mu_n \cosh \mu_n - \sinh \mu_n)}{\mu_n{}^4[\mu_n \cosh \mu_n + \nu_T \sinh \mu_n]} \quad \text{.......} (6\text{-}5.6)$$

where

$$\mu_n = \left[c^2\alpha_n{}^2 + \lambda\tau\right]^{\frac{1}{2}}, \text{ and let } \lambda = i\,\omega \text{ then}$$

$$\frac{F_\beta}{\beta}=\frac{(\omega\tau)^2+i\omega\tau|c\alpha_n|^2}{|c\alpha_n|^4+(\omega\tau)^2}+\frac{Z_5[Z_1Z_4-Z_2Z_3]-i\,Z_5[Z_1Z_3+Z_2Z_4]}{Z_1^2+Z_2^2}$$

.......(6-5.7)

The imaginary part of equation 6-5.7 is the thermoelastoplastic damping coeficient $\left(\frac{G_\beta}{\beta}\right)$ i.e:

$$\frac{G_\beta}{\beta}=\frac{\omega\tau|c.\alpha_n|^2}{|c\alpha_n|^4+|\omega\tau|^2}-\frac{Z_5[Z_1Z_3+Z_2Z_4]}{Z_1^2+Z_2^2} \quad \text{........... (6-5.8)}$$

where

$$\Lambda=\left[(\omega\tau)^2+(n\pi c)^4\right]^{\frac{1}{4}},\quad P=\cos\frac{1}{2}\theta\quad ,q=\sin\frac{1}{2}\theta\qquad ,\theta=\tan^{-1}\left[\frac{\omega\tau}{[c\alpha_n]^2}\right]$$

$$M=\cos\Lambda q.\cosh\Lambda p\quad ,N=\sin\Lambda q.\ \sinh\Lambda P$$

$$\Lambda_1=\left[(n\alpha_n)^4+(\omega\tau)^2\right]^{\frac{5}{4}},\ P_1=\cos\frac{5}{2}\theta\ ,\ q_1=\sin\frac{5}{2}\theta$$

$$R_1=V_T.\left[(c\alpha_n)^4-(\omega\tau)^2\right],\quad R_2=2\omega\tau\,\nu_T.[c\alpha_n]^2$$

$$D_1=\sinh\Lambda P\bullet\cos\Lambda q\ ,\ D_2=\sin\Lambda q\bullet\cosh\Lambda p$$

$$Z_1=\Lambda_1[P_1M-q_1N]+[R_1D_1-R_2D_2]$$

$$Z_2=\Lambda_1[P_1N+q_1M]+[R_1D_2+D_1R_2]$$

$$Z_3=\Lambda[PM-qN]-D_1$$

$$Z_4=\Lambda[PN+qM]-D_2$$

$$Z_5=3(1+V_T)\omega\tau$$

$$G_{\beta O}= \mathrm{Im\,g}[F(\lambda\tau, nC \rightarrow 0, \beta] = \mathrm{Im\,g}[F_0(\lambda T, \beta)]....(7\text{-}1.3)$$

It is important to say that when [nc → o.o], $\xi \rightarrow \infty$ i.e the wavelehgth parameter ξ approaches infinity, which corresponds to the thermal damping coefficient for Bernoulli Euler beam.

Now , three different thermomechanical coupling functions F_β and three thermal damping functions G_β were obtained. These functions are summarized in the following sections.

7-2 ADIABATIC THERMAL DAMPING FUNCTION- $G_{\beta A}$.

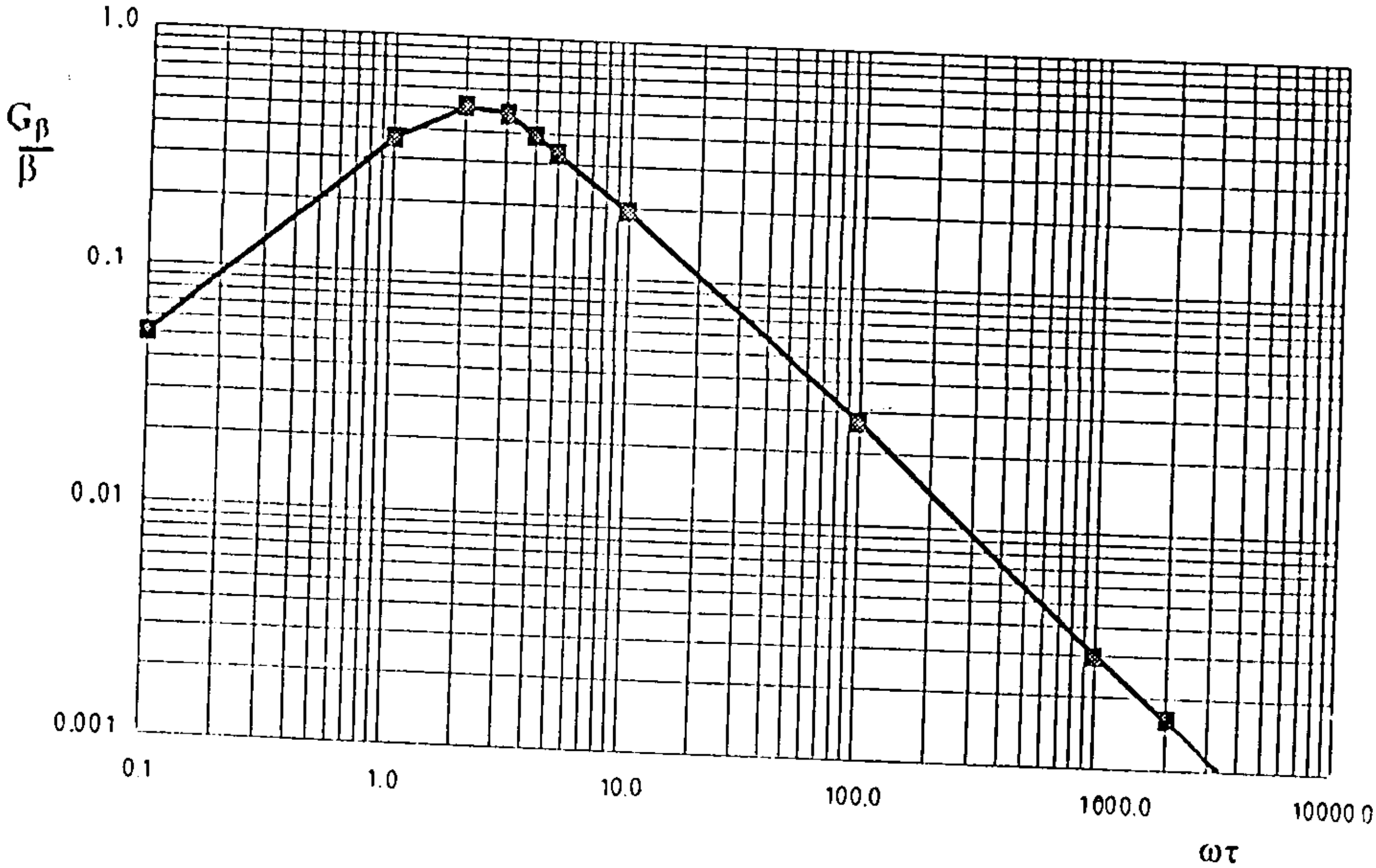
Thermal damping Function $G_\beta(\omega\tau, nc, \beta)$ in which the lateral surfaces of the beam are assumed to be thermally insulated and the end surfaces are assumed to kept at T= o, as nc → o one obtains

$$G_{B_0} = \beta\left[\frac{3.0}{\omega\tau} - \frac{1.0607}{|\omega\tau|^{\frac{3}{2}}} * \frac{\sinh\sqrt{2\omega\tau} + \sin\sqrt{2\omega\tau}}{\left[1 + \sinh^2\sqrt{\frac{\omega\tau}{2}} - \sin^2\sqrt{\frac{\omega\tau}{2}}\right]}\right] ...(7\text{-}2.1)$$

Form equation (7-2.1) one notes that as $\omega\tau >> 1.0$, then

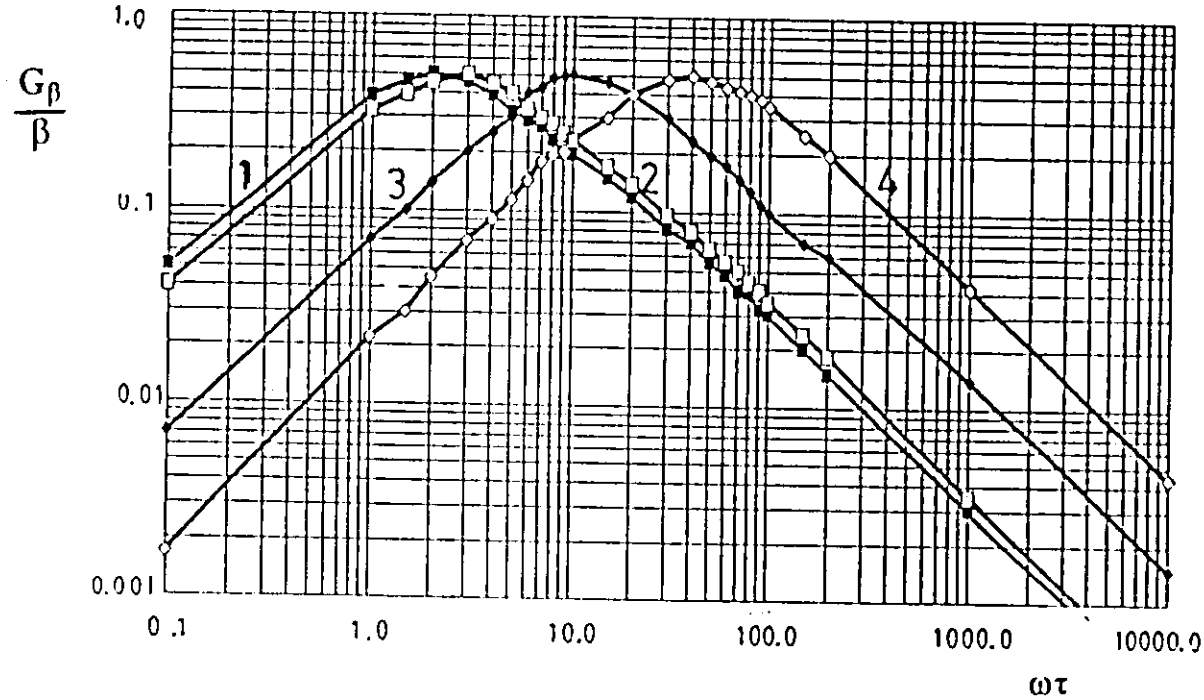
$$G_{\beta_0}= \beta\left[\frac{3}{\omega\tau} - \frac{1.0607}{|\omega\,\tau|^{\frac{3}{2}}} * \frac{\sinh\sqrt{2\omega\tau}}{\sinh\sqrt{\frac{\omega\tau}{2}}}\right] ....(7-2.2)$$

Numerical results for G_β ,$G\beta o$ are presented in Figure (7-2.2) for different values of frequency parameter and for various values of $nc\left(=\frac{1}{g_0}, g_0\right)$ = wave length parameter).



Adiabatic thermal damping coefficient for
Bernoulli - Euler beam i.e nc =o.o .

Figure (7-2.1)



Adiabatic thermal damping coefficient $G_{\beta A}$

where 1 : nc = o   2 : nc = .25

3 : nc = 1   4 : nc = 2

Figuer (7-2.2)

7-3 MIXED THERMAL DAMPING FUNCTION: $G_{\beta M}$ NUMERICAL RESULTS.

In this case one assumes that the coefficients V_T and V_L are as follows:

$$V_T = 1.0 \quad , \quad V_L = 0.0 \qquad \text{..........} (7\text{-}3.1)$$

As an example let us calculate the magnitude of thermoelastoplastic damping function for Euler-Bernouli beam i.e nc → 0.0 From equation (6-3.8) the following parameter values are obtained

$$\theta = \tan^{-1}\left[\frac{\omega\tau}{|n\pi c|^2}\right] = \frac{\pi}{2}$$

$$p = q = \cosh\frac{1}{2}\theta = \frac{1}{\sqrt{2}}$$

$$A = \sqrt{\omega\tau}, \qquad \text{let} \quad Ao = \sqrt{\frac{\omega\tau}{2}}$$

$$M = \cos Ao \cosh Ao \quad , N = \sin Ao \sinh Ao$$

$$A_1 = |\omega\tau|^{\frac{5}{2}} \quad , p_1 = \cos\frac{5}{2}\theta = \frac{-1}{\sqrt{2}}, q_1 = \frac{-1}{\sqrt{2}}$$

$$R_1 = v_T\left[0.0 - (\omega\tau)^2\right] \quad , \quad R_2 = 0.0$$

$$D_1 = \sinh Ao . \cos Ao \qquad D_2 = \sin Ao . \cosh Ao$$

$$Z_1(\omega\tau) = \frac{|\omega\tau|^{\frac{5}{2}}}{\sqrt{2}}[\text{Sin}A_0 \bullet \sinh A_0 - \cos A_0 \bullet \cosh A_0] - [\omega\tau]^2[\sinh A_0 \bullet \cos A_0]$$

$$Z_2(\omega\tau) = \frac{|\omega\tau|^{\frac{5}{2}}}{\sqrt{2}} \bullet [\sin A_0 \bullet \sin A_0 + \text{Cos}A_0 \bullet \text{Cosh}A_0] - [\omega\tau]^2[\sin A_0 \bullet \cosh A_0]$$

$$Z_3(\omega\tau) = \sqrt{\frac{\omega\tau}{2}}[\text{Cos}A_0 \bullet \text{Cosh}A_0 - \sin A_0 \sinh A_0] - \sinh A_0 . \text{Cos}A_0$$

$$Z_4(\omega\tau) = \sqrt{\frac{\omega\tau}{2}}[\sin A_0 \bullet \cosh A_0 - \sin A_0 \sinh A_0] - \sinh A_0 . \text{Cos}A_0$$

$$Z_5(\omega\tau) = 6\omega\tau \text{............} (7-3.2)$$

Substituting into equation 6-3.7 yields:

$$\frac{G_{\beta Mo}}{\beta}=\frac{\omega\tau[n\pi c]^2}{[n\pi c]^2+[\omega\tau]^2}-Z_5(\omega\tau)*\frac{Z_1(\omega\tau).Z_3(\omega\tau)+Z_2(\omega\tau).Z_4(\omega\tau)}{Z_1^2(\omega\tau)+Z_2^2(\omega\tau)}$$

Numerical Example :

Let $\omega\tau = 2.0$ then after substitution through equation (7-3.2) one obtains:

$A_0 = 1.000$ $\quad$ $A_0 = 1.4142$

$M = 0.83370$ $\quad$ $N = 0.9889$

$A_1 = 5.6570$ $\quad$ $P_1 = q_1 = -0.70711$

$R_1 = -4.0$ $\quad$ $R_2 = 0.0$

$D_1 = 0.63496$ $\quad$ $D_2 = 1.2985$

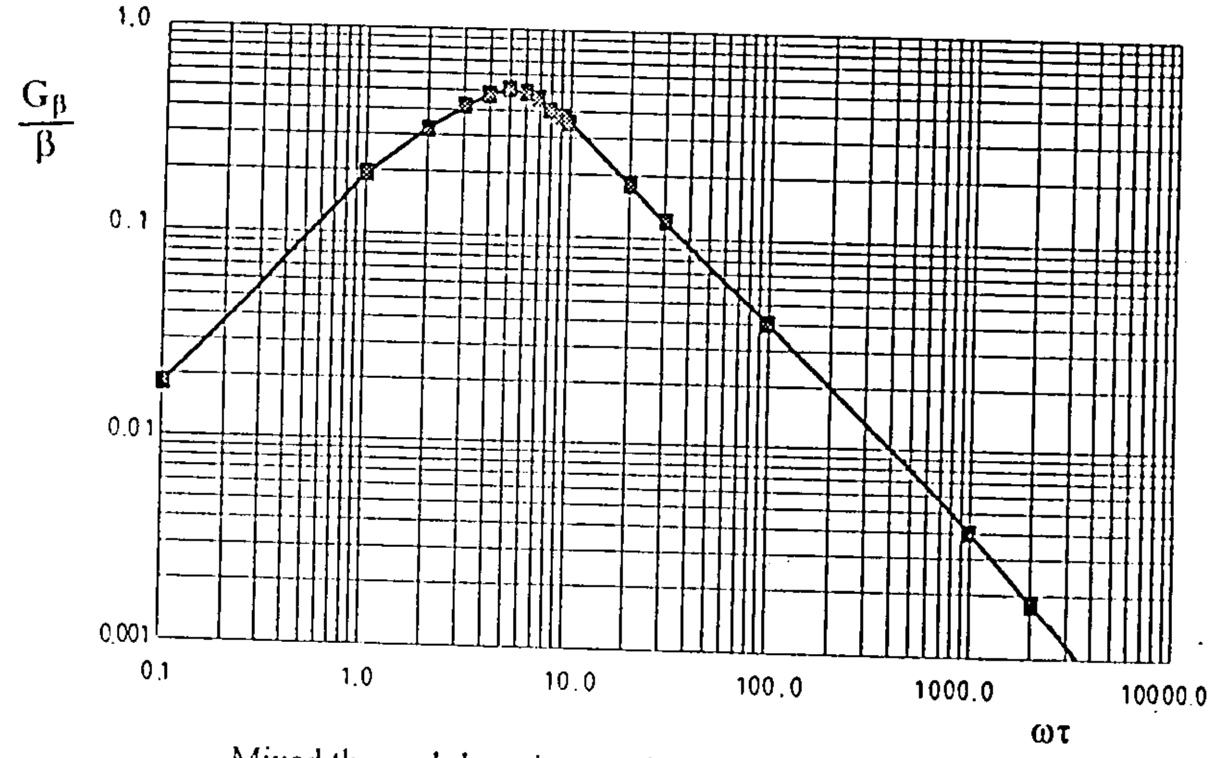
$Z_1 = -1.9190$ $\quad$ $Z_2 = -12.5020$

$Z_3 = 0.79016$ $\quad$ $Z_4 = 0.241$

$Z_5 = 12$

$$\frac{G_{\beta MO}}{\beta} = 0-(-0.3760) = 0.3760$$

Numerical results for $\frac{G_{\beta Mo}}{\beta}$ for different values of $\omega\tau$ are shown in figure (7-3.1) also different values of $\frac{G_{\beta M}}{\beta}$ for different ν values of wavelength parameter are shown in figure (7-3.2).



Mixed thermal damping coefficient for Bernoulli - Euler beam i.e nc =o.o .

Figure (7-3.1)

Figuer (7-3.2)

7-4: ISOTHERMAL THERML DAMPING COEFFICENT $G_{\beta I}$:

NUMERICAL RESULTS

In this case the end and the lateral surfaces of the beam are assumed to be kept at constant temperature i.e :

$$T = o \ \text{at} \ y = \pm a \quad x = o, L \quad ....(7-4.1)$$

This means that the values of the coefficient V_T and V_L can be assumed to be equal i.e:

$$V_T = V_L = 1.0 \ .... (7\text{-}4.2)$$

From equation (6-4.5) one has

$$\tan \alpha_n = 2V_L \cdot \frac{\alpha_n}{\alpha_n^2 - V_L^2} .....(7-4.3a)$$

Now substituting equation (7-4.2) into (7-4.3a) and solving yields

$$\alpha_n \cong 1.305625 \quad .... (7\text{-}4.3b)$$

The general formula for the isothermal thermal damping function $\frac{G_{\beta I}}{\beta}$ is

$$\frac{G_{\beta I}}{\beta} = \frac{\omega\tau |c.\alpha_n|^2}{|c\alpha_n|^2 + |\omega\tau|^2} - 6\omega\tau * \frac{Z_1(\omega\tau).Z_3(\omega\tau) + Z_2(\omega\tau)Z_4(\omega\tau)}{Z_1^2(\omega\tau) + Z^{2(\omega\tau)}_2} ......(7\text{-}4.4)$$

Note that $G_{\beta I}$ is function of three parameters which are:

1- Thermomechanical coupling parameter β.

2- Frequency parameter $(\omega\tau)$.

3- nc where $\xi = \frac{1}{nc}, \xi$ = wave length parameter.

Next as an example the value of $\frac{G_{\beta i_o}}{\beta}$ is calculated for Euler-Berneulli beam i.e nc $\longrightarrow$ o (wave length parameter g_0 infinite).

In this case $c.\alpha_n \longrightarrow o$ **and let** $\omega\tau = 5$ leads to

$P_1 = q_1 = -0.70711 \quad P = q = 0.70711 \quad 0 = \frac{\pi}{2}$

$N = 2.3271 \quad M = -0.02610$

$A_1 = 55.911 \quad R_1 = -25.0$

$R_2 = 0.00 \quad D_1 = -0.0240$

$D_2 = 2.5330 \quad Z_1 = 93.610$

$Z_2 = -154.30 \quad Z_3 = -3.6960$

$Z_4 = 1.1070 \quad Z_5 = 30.00$

Substituting the above results into equation (7.4.4) one obtains

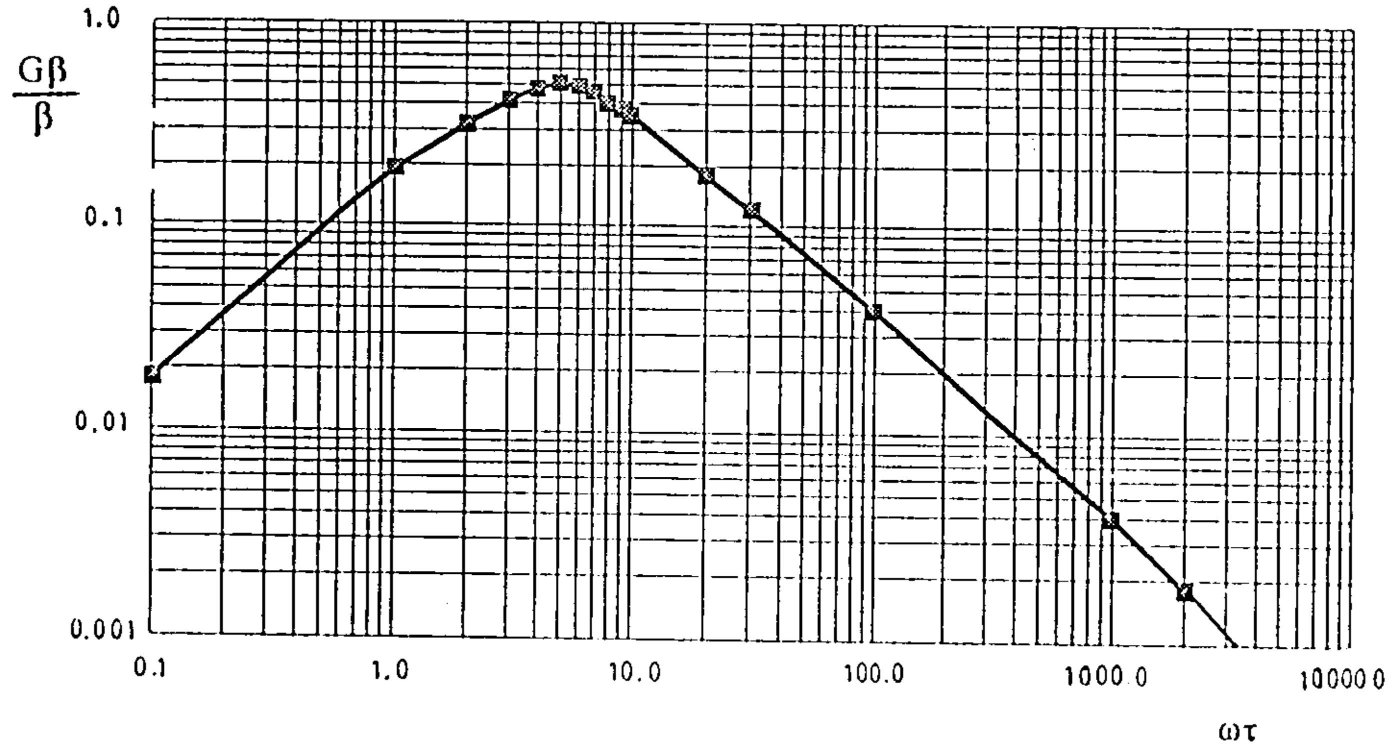
$$\frac{G_{\beta lo}}{\beta} = -6\omega\tau * \frac{Z_1(\omega\tau).Z_3(\omega\tau) + Z_2(\omega\tau)Z_4(\omega\tau)}{Z_1^2(\omega\tau) + Z_2^2(\omega\tau)} \ldots(7-4.5)$$

$$\frac{G_{\beta\, lo(\omega\tau=5)}}{\beta} = 0.4755$$

Numerical results for $\frac{G_{\beta lo}}{\beta}(\omega\tau)$ and $\frac{G_{\beta I}}{\beta}(\omega\tau, nc)$ for various values of frequency parameter $\omega\tau$ and nc are presented in figures (7-4.1) and (7-4.2).

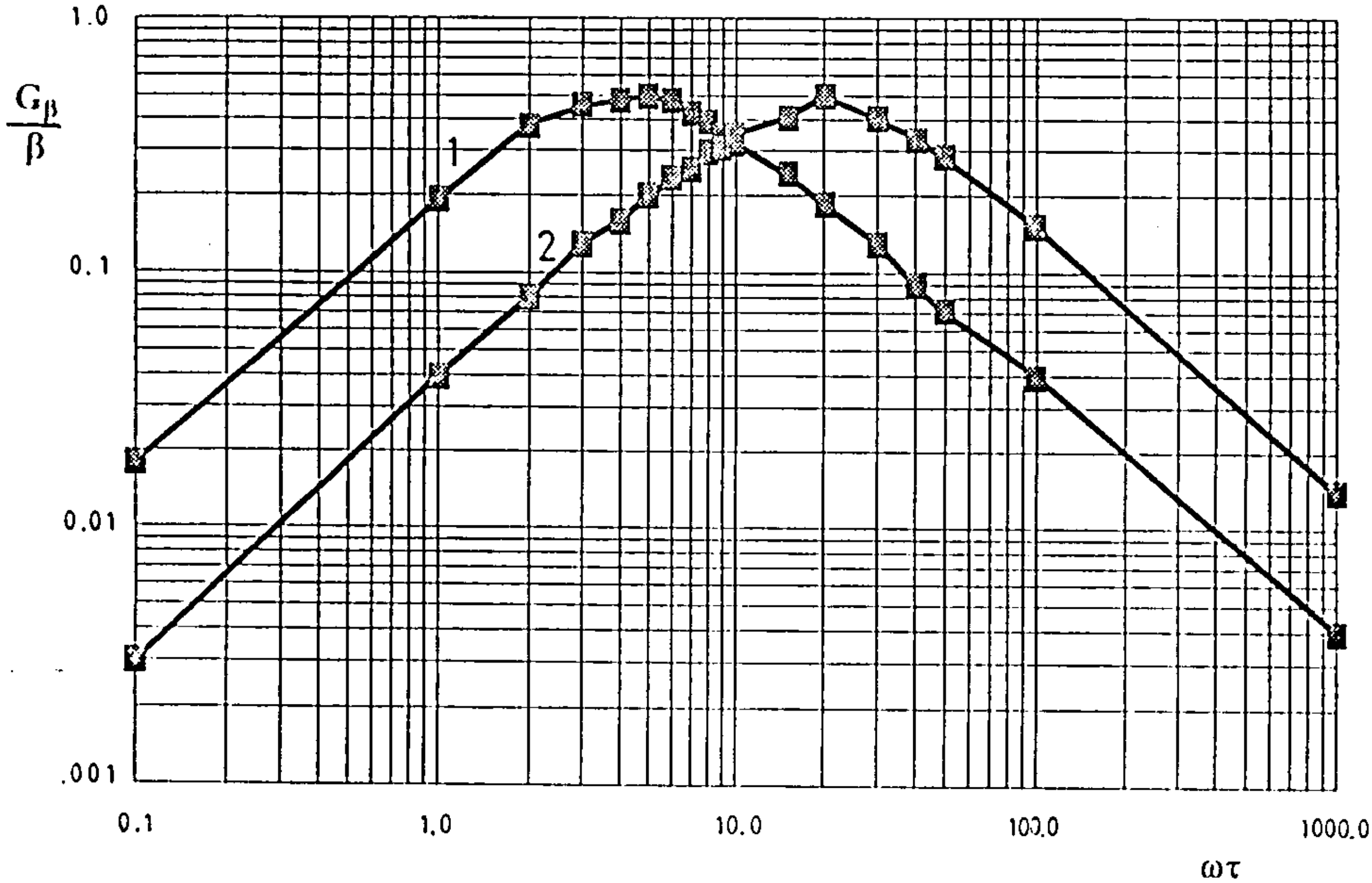
7-5 : NEWTON'S LAW OF HEAT EXCHANG

In this section the magnitude of the thermal damping coefficient under thermal boundary conditions that follow Newton's law of heat exchange is obtained. For the general thermal boundary conditions ,assume that that these boundary conditions obey the Newton surface heat excange law: i.e



Isothermal thermal damping coefficient for
Bernoulli - Euler beam i.e nc =0.0 .

Figure (7-4.1)



Isothermal thermal damping coefficient

1 : α_n= 0.0

2 : α_n= 2

Figure .(7-4.2)

$$K\frac{\partial T}{\partial y} \pm K_T.T = o, \text{at } y = \pm a$$

$$k\frac{\partial T}{\partial X} \pm K_L.T = o, \text{ at } x = L,o \text{..........}(7-5.1)$$

$$\text{or} : \frac{\partial \theta}{\partial \eta} \pm V_T.\theta = o \quad \text{at } \eta = \pm 1$$

$$\frac{\partial \theta}{\partial \zeta} \pm V_L.\theta = o \text{ at } \xi = 1,0 \text{..........}(7-5.2)$$

The solution of the conduction equation under the a bove boundary conditions give :

$$\frac{F_\beta}{\beta} = \frac{\lambda\tau}{\mu_n^2} - \frac{3(1+V_T)\lambda\tau}{\mu_n^4} * \frac{\mu_n \cosh\mu_n - \sinh\mu_n}{\mu_n \cosh\mu_n + V_T.\sinh\mu_n} \text{....}(7-5.3)$$

where:

$$\mu_n = \left[[C\alpha_n]^2 + \lambda\tau\right]^{\frac{1}{2}} \quad , \lambda = i\omega$$

F_β is the thermomechanical coupling function the imaginary part of equation (7-5.3) is the thermal damping function i.e:

$$\frac{G\beta}{\beta} = \frac{\omega\tau[C.\alpha_n]^2}{[C.\alpha_n]^4 + [\omega\tau]^2} - Z_5 * \frac{Z_1.Z_3 + Z_2.Z_4}{Z_1^2 + Z_2^2} \text{....}(7-5.4)$$

Note that tan $\alpha_n = 2V_L.\frac{\alpha_n}{\alpha_n^2 - V_L^2}$ ....(7 – 5.5)

7-6 NEWTON'S LAW : NUMERICAL EXAMPLE

In this section the magnitude of the thermal damping coefficient as functions the Newton's parameter of heat exchange are obtained. As an example let us assume that $V_L = 0.0$, and substituting into equation (7-5.5) yields

$$\tan\alpha_n = o \quad \text{i.e} \quad \alpha_n = n\pi, n = 1,2,3 \text{.......}(7\text{-}6.1)$$

Let $\omega\tau = 5$, $n\pi c = \pi$ i.e $nc = 1.0$ Substituting into equation 7-5-4 one obtains

$$\frac{G\beta}{\beta} = \frac{\omega\tau[n\pi c]^2}{[n\pi c]^4 + [\omega\tau]^2} - \frac{Z_5[Z_1 Z_3 + Z_2 Z_4]}{Z_1^2 + Z_2^2} \quad .... (7\text{-}6.1)$$

$$\theta = \tan^{-1}\left(\frac{\omega\tau}{(n\pi c)^2}\right) = 0.4689$$

$$P = \cosh\frac{1}{2}\theta = 0.97264 \quad , \quad q = \sin\frac{1}{2}\theta = 0.23231$$

$$A = \left[(\omega\tau)^2 + (n\pi c)^4\right]^{1/4}, = 3.3262$$

$$M = \cos Aq * \cosh Ap = 0.716022 * 12.725 = 9.111b$$

$$N = \sin Aq * \sinh Ap = 0.69808 * 12.6859 = 8.8558$$

$$A_1 = \left[(c\alpha_n)^4 + (\omega\tau)^2\right]^{5/4} = 407.162$$

$$P_1 = \cos\frac{5}{2}\theta = 0.3881 \qquad q_1 = \sin\frac{5}{2}\theta = 0.92163$$

$$R_1 = V_T.\left[(cn\pi)^4 - (\omega\tau)^2\right] = 72.41\nu_T$$

$$R_2 = 2\omega\tau . V_T.[c\alpha_n]^2 = 98.696\nu_T$$

$$D_1 = \sinh Ap * \cos Aq = 9.0834$$

$$D_2 = \sin Aq * \cosh Ap = 8.8831$$

$$Z_1 = A_1.[P_1.M - q_1.N] + [R_1.D_1 - R_2.D_2] = -1883.35 - 218.9\nu_T$$

$$Z_2 = A_1.[P_1.N + q_1.M] + [R_1.D_2 + D_1.R_2] = 4188.5 + 1539.7\nu_T$$

$$Z_3 = 13.55 \qquad Z_4 = 26.811$$

$$Z_5 = 15(1 + V_T)$$

Now substituting into equation (7-6.1) one gets the value of thermal damping as function of ν_T at $\omega\tau = 5$:

$$\frac{G\beta}{\beta} = 0.40314 - \frac{57.47\nu_T^2 + 212.96\,\nu_T + 155.5}{241.86\nu_T^2 + 1566.25\nu_T + 2676.5} \quad .... 7\text{-}6.2$$

Now let $\omega\tau = 25$, $nc = 1$, after substituting in equation (7-6.1) one gets the value of the thermal damping as a function of V_T as:

$$\frac{G_{\beta}(\nu_T)}{\beta} = 0.3415 + \frac{-1.097\nu_T^2 + 0.295\nu_T + 10.39}{6.89\nu_T^2 + 59.12\nu_T + 185.4} \quad \text{..... 7-6.3}$$

Also let $\omega\tau = 70$ we get:

$$\frac{G_{\beta}}{\beta} = 0.1382461 + \frac{3.87\nu^2_T + 5462\nu_T + 5153}{6.89\nu_T^2 + 28508\nu_T + 15398} \quad \text{...... 7-6.4}$$

Numerical results for the values of thermal damping coefficient as function of V_T is shown in Figure (7-6.1).

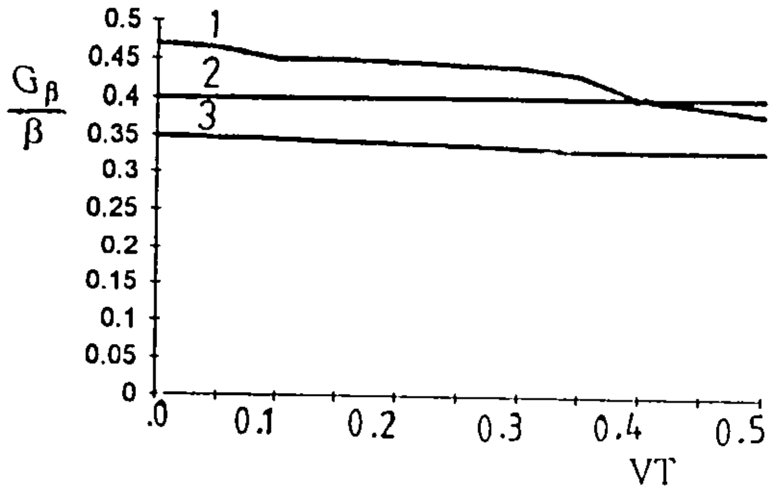
7-7 DISCUSSION AND RESULTS

Within the context of the linear thermoelasticity theory including thermomechanical coupling effect, the free vibrations of rectangular cross section beam, and then thermoelastoplastic damping coefficient functions are studied. The surfaces of the beam are either kept at constant temperature (isothermal) or thermally insulated (adiabatic), or the end surfaces are thermally insulated and the lateral surfaces are kept at constant temperature (mixed thermal boundary condition).

The govering equations are derived for the case of general thermal boundary conditions that follows the Newton's surface heat transfer law.

The thermomechanical coupling function F given by equations (6-2.6),(6-3.4), (6-4.6), represents the effect of thermomechanical coupling on the dynamic system. The real part of this function characterize the stiffening effect while the imaginary part represents thermal damping effect. Thus it is important to say that for rectangular cross sectional beams the thermoelasto plastic damping coefficient is a function of the frequency parameter $\omega\tau$, wave lenth parameter $\xi \equiv \frac{\lambda}{nc}$ and the thermomechanical coupling parameter β. The limiting case is

$$G_{\beta o}(\omega\tau,\beta) = \mathrm{Im}\{Fo(\lambda\tau,\beta)\}$$



Variation of the thermal damping coefficient as function of Newton heat exchange parameter VT

1 : $\omega\tau = 70$. 2 : $\omega\tau = 25$ 3 : $\omega\tau = 5$

Figure (7-6.1)

of G_β as nc approaches zero, i.e the wave lenth parameter ξ approaches infinity is the corresponding damping coefficient for Bernoulli-Euler beam.

It should be noted that Zener [4] has obtained the thermoelastic damping coefficient corresponding to Go in a series form in which ($\lambda = i\omega$), λ is a an eigenvalue of the charecteristic equation.

The adiabatic thermal damping coefficient $G_\beta A/_\beta$ is plotted as a function of frequency parameter $\omega\tau$ and wave lenth parameter $\xi \equiv \frac{1}{nc}$. These curves are characteristic curves and independent of beam material properties. The difference in ordinates between $\frac{G_{\beta Ao}}{\beta}$, $\omega\tau$ curve and any other curves for $\xi < \infty$ in these figures represents the contribution of longitudinal heat flows on the thermoelastoplastic damping coefficient (contributed by transverse heat flows) for that particular value of ξ. The contribution of this difference for small values of nc (large values of ξ) is found to be negligibly small. But this contribution becomes more important as nc values become increases.

Also from the obtained Figures it is observed that the maximum values of the thermal damping function $\frac{G_{\beta A}}{\beta}$ are almost identical for all values of ξ. Therefore, the main effect of longitudinal heat flows on the thermal damping coefficient functions is to shift the $G_{\beta Ao}(\omega\tau)$ curves in parallel toward the direction of increasing $\omega\tau$.

Also shown in figure (7-7.2) the thermal damping coefficient as function of $\omega\tau$ governed by Zener's approximate formula:

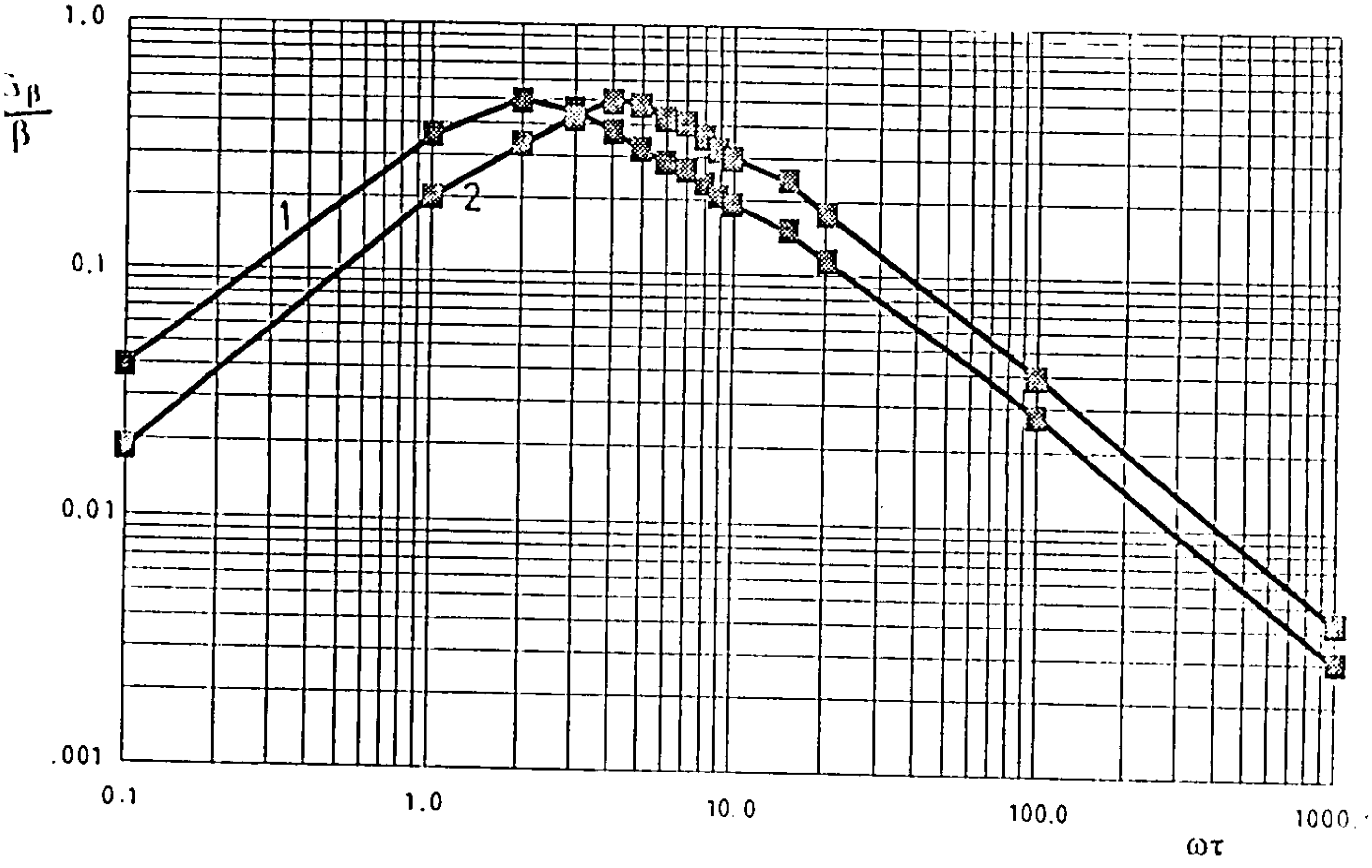
$$\frac{G_{Z\beta}}{\beta} = 0.988\frac{k_m \cdot \omega\tau}{1+(k_m\omega\tau)^2}(k_m = 0.295)$$

This formula is seen to approximate the damping coefficient $\frac{G_{\beta o}}{\beta}(\omega\tau)$ quite well where for large values of $\omega\tau$ the error becomes larger as $\omega\tau$ becomes smaller. For example at

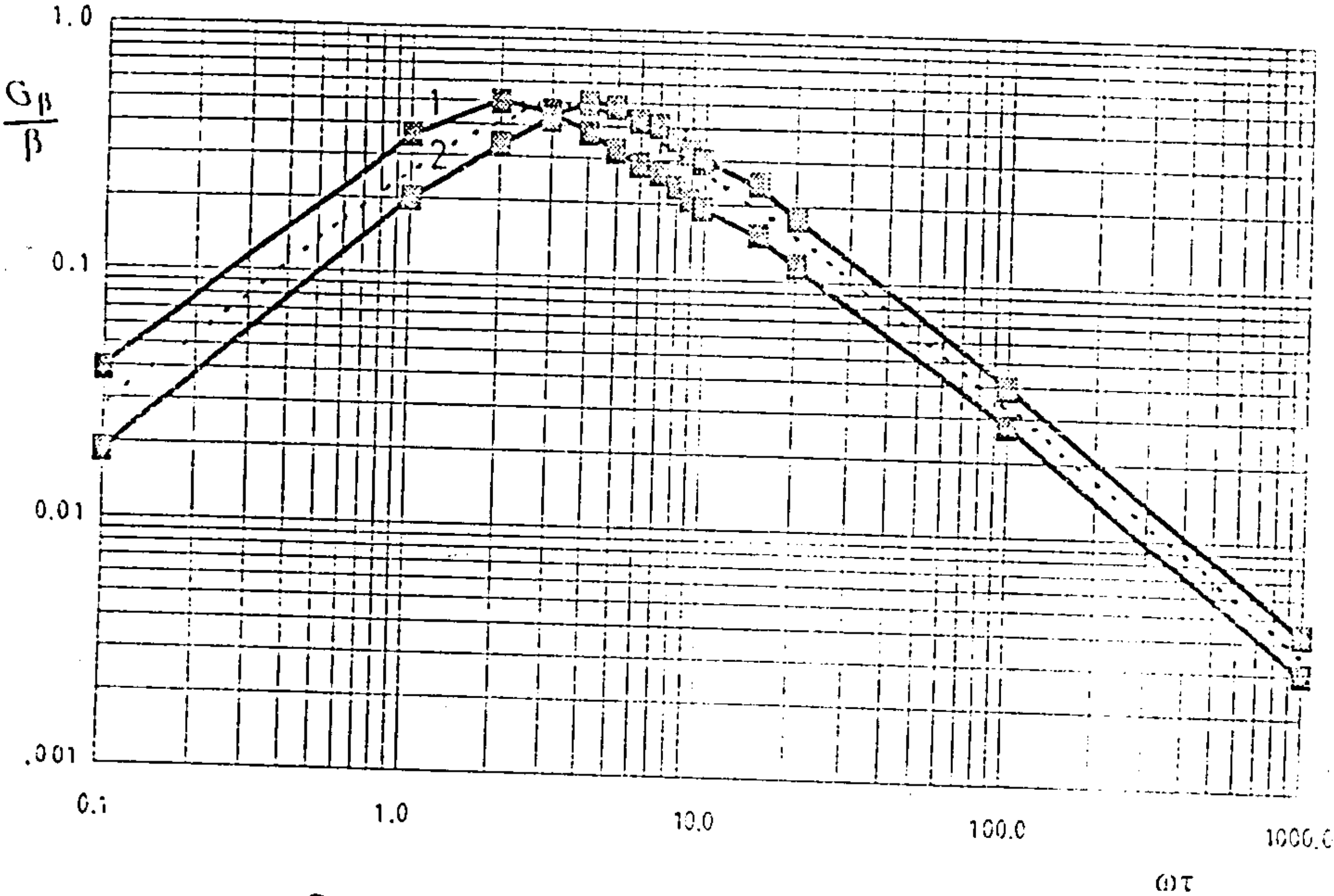
$$\omega\tau = 10^{-1}, \frac{G_{Z\beta}}{\beta} = 3 \times 10^{-2}, \frac{G_{\beta o}}{\beta}\Big|_{exact} = 5 * 10^{-2}, \text{the error} = 2 * 10^{-2}$$

error percent = 40%

The very small thermal damping coeffiecient when the frequency is very small means that the coupling term in the heat conduction equation is very small and also the magnitude of the temperature gradient is very small, and therefore the heat generated due to the temperature change is very small.



Comparison between $G_{\beta Mo}$, $G_{\beta Mo}$, $G_{\beta Mo}$
For Bernulli - Eulear beam
1 : Adiabatic 2 : Iso thermal and Mixed
Figure (7 - 7.1)



Comparison between adiabatic , Iso thermal - thermal damping Coefficient and Zener formula - For Bernoulli - Euler beam - i.e nc = o.o , where
1 : Adiabatic 2 : Iso thermal
..... Zener

Figure. (7- 7.2)

8- The contribution of the longitudinal heat flows on the thermal damping coefficient functions $\frac{G_\beta}{\beta}$ is negligibly small for a large value of wave length parameter ξ i.e (small value of ne). But this contribution becomes more important as the value of g_0 becomes smaller.

9- Temperature change due to plastic deformation is found to be greater than that in elastic deformation.

10- For lower frequencies the magnitude of the adiabatic thermal damping coefficient is greater than the isothermal thermal damping coefficient.

11- For higher frequencies the magnitude of the adiabatic thermal damping coefficient is lower than the isothermal thermal damping coefficient.

12- Temperature varied symmetrically around the axis of symmetry of the beam for Bernoulli - Eulear beam.

13- The magnitude of the temperature change for large frequencies is small, and this change is independent of the frequency.

7-9 RECOMMENDATIONS:

1- In addition to the cases studied in the preceeding chapters the two way coupled heat conduction equation can be solved and the temperture can be taken as a variable and thus the mechanical term in this equation can be considered as function of temperature.

2- The isothermal boundary condition can ben considered to take a value other than zero i.e T=To and then the heat conduction equation can be solved under this general isothermal boundary conditions.

3- For elastic perfectly plastic medium the heat supplied to the system i.e ρR can be considered as function of temperature and strain and thus can be considered to take a value other than zero.

4- The stored elastic energy, was considered as a fucntion of stress and strain only. But since the strains are function of temperature one can considere the stored energy as function of temperature also.

5- Carefully designed experimental set-ups should be considered to check the validity and accuracy of the derived theoretical models.

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```
C      THIS IS A PROGRAM TO CALCULATE THE MAGNITUDE OF
C      THE THERMOELASTO-PLASTIC DAMPING COEFICIENT C
       GB CORRESPONDING TO MIXED, ADIABATIC,
C      ISOTHEMAL BOUNDARY CONDITIONS.

OPEN (1,FILE = ADAB.OUT', STATUS =' NEW')
WRITE (*,*)' ENTER THE VALUE OF WT'
READ (*,*) WT
WRITE (*,*)' ENTER THE VALUE OF NPC'
READ (*,*) NPC
WRITE (*,*)' ENTER THE VALUE OF A'
READ (*,*) A
WRITE (*,*)' ENTER THE VALE OF P'
READ (*,*) P
WRITE (*,*) 'ENTER THE VALUE OF q'
READ (*,*) q
WRITE (*,*) 'ENTER THE VALE OF THETA'
READ (*,*) TH
WRITE (*,*)' ENTER THE VALUE OF PI'
READ (*,*) PI
WRITE (*,*)' ENTER THE VALUE OF ql'
READ (*,*) ql
WRITE (*,*)' ENTER THE VALUE OF VT'
READ (*,*) VT
WRITE (*,*)' ENTER THE VALUE OF AI'
READ (*,*) AI
FO1 = F1 (WT,NPC)
GO1 = G1 (WT,NPC)
FO2 = F2 (WT,NPC)
GO2 = G2 (WT,NPC)
```

```
FO3 = F3 (WT,NPC)
GO3 = G3 (WT,NPC)
FO4 = F4 (WT,NPC)
GO4 = G4 (WT,NPC)

FO5 = F5 (WT,NPC)
FO5 = G5 (WT,NPC)
FO6 = G6 (WT,NPC)
GO6 = G6 (WT,NPC)

WRITE (*,9) GO6
WRITE (1,9) GO6
FORMAT (6X, I3, F12.7)
```

$$GO6 = \frac{(WT)*(NPC)**2.}{(NPC)**4.+(WT)**2.} - \frac{FO6*(FO4*FO5+GO4*GO5)}{(FO4)**2+(GO4)**2}$$

```
END
```

```
FUNCTION F1(WT,NPC)
F1 = COS (A * q) * COSH(A * p)
RETURN
END

FUNCTION G1 (WT,NPC)
G1 = SIN (A * q) * SINH (A * q)
RETURN
END

FUNCTION F2 (WT,NPC)
F2 = VT * ((NPC) **4. - (WT)**2.)
RETURN
END

FUNCTION G2 (WT,NPC)
G2 = 2. * WT * VT * (NPC) * * 2.
RETURN
END

FUNCTION F3 (WT, NPC)
F3 = SINH (A * q) * Cos (A * q)
RETURN
END

FUNCTION G3 (WT, NPC)
G3 = SIN (A * q) * CosH (A * q)
RETURN
END
```

```
FUNCTION F4 (WT, NPC)
F4 = A1 * (P1 * FO1 - q1 * GO1) + (FO2 * FO3 - GO2 * GO3)
RETURN
END

FUNCTION G4 (WT, NPC)
G4 = A1 * (P1 * GO1 + q1 * FO1) + (FO2 * GO3 + FO3 * GO2)
RETURN
END

FUNCTION F5 (WT, NPC)
F5 = A * (P * FO1 - q * GO1) - FO3
RETURN
END

FUNCTION G5 (WT,NPC)
G5 = A * (P * GO1 + q * FO1) - GO3
RETURN
END

FUNCTION F6(WT, NPC)
F6 = 3 * (1. + VT) * WT
RETURN
END
```

```
C       THIS IS A PROGRAM TO CALCULATE THE MAGNITUDE OF
C       THE NORMALIZED TEMPERATURE    NO = GO6
C       AS FUNCTION OF THE NORMALIZED COORDINATE  Y.

        OPEN (1, FILE =' TDCF. OUT', STATUS =' NEW')
        WRITE (*,*)' ENTER THE VALUE OF Y'
        READ (*,*) Y
        WRITE (*,*)' ENTER THE VALUE OF S'
        READ (*,*) S
        FO1 = F1 (Y, S)
        GO1 = G1 (Y, S)
        FO2 = F2 (Y, S)
        GO2 = G2 (Y, S)

        FO3 = F3 (Y, S)
        GO3 = G3 (Y, S)
        FO4 = F4 (Y,S)
        GO4 = G4 (Y,S)

        FO5 = F5 (Y,S)
        GO5 = G4 (Y,S)
        GO6 = G6 (Y,S)
        WRITE (*, 9) GO6
        WRITE (1, 9) GO6
      9 FORMAT (6X, I3, F 12.7)
        GO6 = ((2.0 * Y-FO5)**2. + GO5**2.)**0.5
        END
```

```
FUNCTION F1 (Y,S)
F1 = SINH (0.5 * S) * COS (0.5 * S)
RETURN
END

FUNCTION G1 (Y, S)
G1 = COSH (0.5 * S) * SIN (0.5 * S)
RETURN
END

FUNCTION F2 (Y, S)
F2 = SINH (S) * COS (S)
RETURN
END

FUNCTION G2 (Y,S)
G2 = COSH (S) * SIN (S)
RETURN
END

FUNCTION F3 (Y,S)
F3 = SINH (S *  Y) * COS (S *  Y)
RETURN
END

FUNCTION G3 (Y, S)
G3 = COSH (S * Y) * SIN (S * Y)
RETURN
END
```

```
FUNCTION F4 (Y, S)
F4 = FO2* (FO1 * FO2-GO1 * GO3) + GO2* (GO1 * FO3 + FO1*GO3)
RETURN
END

FUNCTION G4 (Y, S)
G4 = FO2 * (GO1* FO3 + FO1* GO3) + GO2*(GO1* GO3 - FO1* FO3)
RETURN
END

FUNCTION F5 (Y,S)
F5 = 2 * (FO4 + GO4)/(S * (FO2 ** 2. + GO2 ** 2.))
RETURN
END

FUNCTION G5 (Y,S)
G5 = (FO4 - GO4)/FO2 **2. + GO2 ** 2.)
RETURN
END
```

```
C      THIS IS A PROGRAM TO CALCULATE THE MAGNITUDE OF
C      THE PHASE ANGEL  THEATA = GO6  AS FUNCTION OF THE
C      NORMALIZED COORDINATE  Y  IN  RADIANS.

OPEN (1, FILE =' THETA. OUT', STATUS =' NEW')
WRITE (*,*)' ENTER THE VALUE OF Y'
READ (*,*) Y
WRITE (*,*)' ENTER THE VALUE OF S'
READ (*,*) S
FO1 = F1 (Y, S)
GO1 = G1 (Y, S)
FO2 = F2 (Y, S)
GO2 = G2 (Y, S)

FO3 = F3 (Y, S)
GO3 = G3 (Y, S)
FO4 = F4 (Y,S)
GO4 = G4 (Y,S)

FO5 = F5 (Y,S)
GO5 = G4 (Y,S)
GO6 = G6 (Y,S)
WRITE (*, 9) GO6
WRITE (1, 9) GO6
9 FORMAT (6X, I3, F 12.7)
GO6 = ARCTAN (FO5/GO5)
END
```

الملخص

الحساب النظري لمعامل الإخماد الحراري اللدن المرن في الجيزان ذات المقطع المستطيل

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إشراف: د. محمد نادر حمدان

تحظى ظاهرة حساب معاملات الاخماد Damping Coefficients اللدنة والمرنة في الجيزان على إختلاف اشكالها باهتمام كبير وذلك نظراً لأهمتيها العملية والفيزيائية وفي تصميم عدد كبير من المنشآت والاشكال الهندسية. إلا إن ظاهرة حساب معامل الاخماد الحراري (Thermal Damping) في الاشكال الهندسية بقيت مهملة حتى وقت قريب. ومن هنا نجد ندرة كبيرة بالابحاث المتعلقة بحساب الحرارة ومعامل الاخماد الحراري الناتجة عن الاهتزازات المستعرضة والطوليه في الجيزات المستطيلة المقطع. وفي هذا البحث واعتماداً على المبادئ المستخلصة من نظرية المرونة واللدونة والقوى الحرارية و نظرية إنتقال الحرارة في الاجسام الصلبة تم بناء نموذج رياضي مفصل لحساب الاجهادات الحرارية [Thermal Stresses and Strains] والتي من خلالها تم التوصل إلى النموذج الرياضي اللازم لوصف وحساب ظاهرة الاخماد الحراري (Thermal Damping) في الجيزان المستطيلة المقطع في الحالات التالية:

١- كل الأوجه معزولة حرارياً: Adiabatic Bc's

٢- كل الأوجه محافظة على درجة حرارة ثابتة: Isothermal Bc's

٣- مزيج من الحالتين السابقتين Mixed Bc's

وعند المقارنة بين النتائج التي تم التوصل إليها في هذا البحث وما تم التوصل إليه من قبل باحثين مثل (Zener 1937) نجد أن النتائج التي تم التوصل إليها هنا تظهر قدرة كبيرة على تمثيل هذه الظاهرة بشكل دقيق وملائم. كما أجريت بعض التحليلات الرياضية في طيات هذه الدراسة بحيث تم التوصل في النهاية إلى نموذج رياضي مفصل لحساب معامل الاخماد الحراري في الحالات الثلاثة المذكورة سابقاً. وفي النهاية تم تقديم عدد من التوصيات للمتابعة في دراسة هذه الظاهرة.